Pulling Out the Stops: Designing a Muscle for a Tug-of-War Competition

**Objective**

The force-producing properties of muscle are complex, highly nonlinear, and can have substantial effects on movement (see McMahon, 1984 for a review). For simplicity, lumped-parameter, dimensionless muscle models that are capable of representing a variety of muscles with different architectures are commonly used in the dynamic simulation of movement (Zajac, 1989). In this exercise, you will explore the differential equations that describe muscle activation and muscle–tendon contraction dynamics when using a Hill-type muscle model. You will use OpenSim to implement a simple muscle–tendon model and conduct simulations to investigate how various model parameters affect the dynamic response of the actuator. The lab will conclude with a Virtual Muscle Tug-of-War in which you will design an optimal muscle and compete against other user-generated muscles. May the best muscle win!

By working through this lab, you will:

- Become familiar with the differential equations describing muscle activation and muscle–tendon contraction dynamics.
- Learn how to model and simulate muscle–tendon dynamics using OpenSim.
- Become comfortable with modifying existing code that models muscle activation and muscle–tendon mechanics.
- Explore the effect of various model parameters and simulation conditions on the dynamic response of muscle.
- Design your own optimal muscle–tendon actuator to compete in a virtual muscle tug-of-war.

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**Model**

The model in this exercise consists of a cube with a single translational degree of freedom along the Z-axis. A Tug_of_War model has been included in the OpenSim distribution (Models/Tug_of_War/) that uses two Thelen 2003 Muscles to move the cube. In this exercise, we will use two Millard 2012 Muscles instead of the Thelen muscles. The muscles are arranged to pull on opposite sides of the cube. The cube has a mass of 20 kg and sides of length 0.1 m, and the distance between the fixed ground supports is 0.7 m. Thus, each muscle–tendon actuator is 0.3 m long when the block is centered.

**Background**

**Millard 2012 Muscle Model**

Musculotendon actuators consist of an active contractile element, a passive elastic element, and an elastic tendon. The maximum active force a muscle can develop varies nonlinearly with its length, represented by the active-force–length curve.
, peaking at a force of 

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at a length of 

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(the tilde is used to denote forces, velocities, muscle lengths, and tendon lengths that are normalized by 

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, and 

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, respectively). During non-isometric contractions, the force developed by muscle varies nonlinearly with its rate of lengthening, which is represented by the force–velocity curve 

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. Force is also developed when muscle is stretched beyond a threshold length, regardless of whether the muscle is activated, which is represented by the passive-force–length curve 

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. Muscle force ( 

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) is computed using these curves as follows:
where

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is the muscle activation, which ranges from

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to 1.

Muscle attaches to bone through tendon. Since a long tendon may stretch appreciably beyond its slack length ()

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\) when under tension, tendon is modeled as a nonlinear elastic element developing force according to the tendon-force–length curve

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. Muscle fibers attach to tendon at a pennation angle ()

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\), scaling the force they transmit to the tendon. If the tendon is assumed to be elastic and the mass of the muscle is assumed to be negligible, then the muscle–tendon force equilibrium equation (i.e.,

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\) takes the following form:

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We have developed default force curves for the musculotendon model that have been fit to experimental data (see Millard et al., 2013 for details). These curves can be adjusted to model muscle and tendon whose characteristics deviate from these default patterns.

In a forward-dynamic simulation, the force generated by a musculotendon actuator is calculated from the length (}
Equation (1) alone cannot be used to solve for the muscle force because multiple combinations of muscle length and velocity will satisfy the equation. A unique solution for $\theta$, velocity ($v$), and activation ($a$) can be found by solving Eq. (1) for the normalized muscle velocity ($\tilde{v}$) to obtain an ordinary differential equation, which can then be integrated to simulate muscle contraction (see Zajac, 1989):

$$\frac{d\tilde{v}}{dt} = \frac{1}{\tilde{F}(\theta)}$$

where $\tilde{F}(\theta)$ is the inverse of the force–velocity curve. Although Eq. (1) is devoid of numerical singularities, Eq. (2) has four as
Since these conditions are often encountered during a simulation, the quantities causing singularities in Eq. (2) are altered so that the singularities are approached but never reached:

Without modifying the formulation of the equilibrium model, the muscle is able to reach unrealistically short lengths (Thelen, 2003) and cannot be simulated when fully deactivated. We use a unilateral constraint on muscle length to prevent the muscle from becoming unrealistically short:

where

is a candidate value for

computed using Eq. (2).
Design Challenge

The competition is a single-elimination tournament between pairs of muscles. Each match is a one-second forward dynamic simulation where each muscle starts with minimal activation ($a = 0.01$). The match is won by the muscle that has moved the block to its side of the arena at the end of the simulation. Your challenge is to specify the muscle–tendon parameters and excitation time history necessary to overcome the other challengers.

References