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A PLANAR MODEL OF THE KNEE JOINT TO CHARACTERIZE THE KNEE EXTENSOR MECHANISM

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Abstract—A simple planar static model of the knee joint was developed to calculate effective moment arms for the quadriceps muscle. A pathway for the instantaneous center of rotation was chosen that gives realistic orientations of the femur relative to the tibia. Using the model, nonlinear force and moment equilibrium equations were solved at one degree increments for knee flexion angles from 0 (full extension) to 90°, yielding patellar orientation, patellofemoral contact force and patellar ligament force and direction with respect to both the tibial insertion point and the tibiofemoral contact point.

The computer-derived results from this two-dimensional model agree with results from more complex models developed previously from experimentally obtained data. Due to our model's simplicity, however, the operation of the patellar mechanism as a lever as well as a spacer is clearly illustrated. Specifically, the thickness of the patella was found to increase the effective moment arm significantly only at flexions below 35° even though the actual moment arm exhibited an increase throughout the flexion range. Lengthening either the patella or the patellar ligament altered the force transmitted from the quadriceps to the patellar ligament, significantly increasing the effective moment arm at flexions greater than 25°. We conclude that the levering action of the patella is an essential mechanism of knee joint operation at moderate to high flexion angles.

NOMENCLATURE

θ	flexion of the femur relative to the tibial axis (degrees)
θ_q	quadriceps-force angle with respect to the tibial axis (degrees)
α	patellar axis angle with respect to the tibial axis (degrees)
β	patellar ligament angle with respect to the tibial axis (degrees)
t	distance from longitudinal axis of patella to its posterior surface (cm)
l_p	patellar length (cm)
l_{pl}	patellar ligament length (cm)
F_q	force applied by the quadriceps (N)
F_r	patellofemoral joint reaction (compression) force (N)
F_{pl}	patellar ligament force (N)
M_q	moment arm of quadriceps about patellofemoral contact point (cm)
M_{pl}	moment arm of patellar ligament about patellofemoral contact point (cm)
M_{act}	actual moment arm of patellar ligament about tibiofemoral contact point (cm)
M_{eff}	effective moment arm (cm)
TF	tibiofemoral
PF	patellofemoral

INTRODUCTION

Knowledge of joint moment arms is useful to the study of human movement. With such information, researchers may compute the moment of muscle force about a joint. However, because of its three-bone, multiligamented structure, the knee provides a challenge to the determination of the moment arm for the quadriceps. In this study, we developed a simple model of the knee that allows the essence of knee mechanics to be clearly understood. The model can be used to calculate the extensor moment arm with a minimum of computation, so that it can be used in whole-body dynamic simulations of motion without consuming a disproportionate share of computational resources.

One of the classic studies on the subject of knee moment arms was performed by Smidt (1973), in which the method of Reuleaux was used to determine the location of the instantaneous center of joint rotation (ICR) at selected flexion angles. He assumed that the moment arm was the perpendicular distance from the ICR to the central axis of the extensor force acting through the patellar ligament. Additionally, the patella was treated as a device which served to redirect the force exerted by the quadriceps around the distal femur. Thus, tensions in the patellar ligament and quadriceps tendon were treated as being equal in magnitude. The extensor muscle moment at the tibiofemoral joint and patellofemoral compression force were also calculated.

Gertzbein *et al.* (1985), Bryant *et al.* (1984) have, however, delineated weaknesses and suggested improvements. Soudan *et al.* (1979) have suggested the more accurate 'tangent method' instead of the method of Reuleaux, to locate the ICR pathway. Others have stressed the importance of including the tibiofemoral contact forces in the calculation of the extensor muscle moment, if moments are summed about the ICR (Reilly and Martens, 1972; Grood *et al.*, 1984; Nisell *et al.*, 1986).

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Denham and Bishop (1978) found that the forces in the quadriceps tendon and patellar ligament are unequal in magnitude. Goodfellow *et al.* (1976) observed that the patella's region of contact with the femur formed a narrow band of contact stretching mediolaterally across the posterior surface of the patella and moved proximally along the patella as the knee flexed. Thus the patella seems to act not only as a spacer to increase the moment arm of the joint (Kaufer, 1971), but also as a lever to alter the magnitude, as well as the direction, of the force transmitted from the quadriceps tendon to the patellar ligament.

Recognizing that patellar leveraging directly affects the muscle moment at the tibiofemoral joint, Grood *et al.* (1984) defined an 'effective' moment arm that expresses the extensor moment arm in terms of the quadriceps force; *i.e.* the effective moment arm is the product of the actual moment arm (perpendicular distance from the patellar ligament to the tibiofemoral contact point) and the ratio of force in the patellar ligament to the force in the quadriceps tendon. Their curves of effective moment arm exhibit sharply peaked maxima centered at 20–30° of flexion, whereas previously-reported moment arms peaked less sharply at 40–50°. However, very small effective moment arms were found at large flexion angles, presumably in conflict with some of the extensor muscle moments reported by other researchers (Lindahl *et al.*, 1969; Scudder, 1980).

More recently, van Eijden *et al.* (1985) found that the sagittal-plane angle between the distal part of the quadriceps tendon and the femoral axis varies as the knee is flexed. We used the model, therefore, to see if the direction of the force applied by the quadriceps is as important as the levering action of the patella in affecting the effective moment arm.

DEVELOPMENT OF A SIMPLE KNEE MODEL

Background

Many investigators have utilized planar models of the knee (Moeinzadeh *et al.*, 1983; Wongchaisuwat *et al.*, 1984; Nisell *et al.*, 1986). These sagittal-plane models are often justifiable because axial rotation in the joint is less than 10° and becomes noticeable only near full extension (Hallen and Lindahl, 1966; Markolf *et al.*, 1976). Rocking motions of the patella are also approximately planar, as they occur about a fulcrum stretching medio-laterally, approximately perpendicular to the plane of action of the extensor tendons and ligaments (Hungerford and Barry, 1979).

For a planar model, two-dimensional motion descriptors such as the ICR pathway can be used to define a planar approximation to the three-dimensional motion. Though the geometry of the articulating surfaces (Blacharski *et al.*, 1975; Murphy *et al.*, 1985), muscular geometry and forces, and the restraining action of the cruciates and ligaments (Morrison, 1970; Wongchaisuwat *et al.*, 1984) may all contribute to producing the ICR pathway, it is attractive as a simple, qualitative descriptor of the instantaneous motion of the tibiofemoral joint (Frankel and Burstein, 1970). Conversely, a *prescribed* ICR pathway, if chosen judiciously, can be used in a simple tibiofemoral model to impose realistic, macroscopic behavior without requiring the detailed geometric descriptions of the joint and ligamentous system characteristic of more complex models (Andriacchi *et al.*, 1983; Crowninshield *et al.*, 1976; Wismans *et al.*, 1980). Since it seems that the femoral condyles approach rolling near full extension, and slip increasingly on the tibial plateau as the joint is flexed (Lindahl and Movin, 1967; Moeinzadeh *et al.*, 1983; Wongchaisuwat *et al.*, 1984), the ICR pathway should be nearly coincident with the tibiofemoral contact point at full extension.

Using an ICR pathway to represent the motion of the tibiofemoral joint in two dimensions presumes that the geometry of the tibia and femur can be adequately represented in a planar fashion. In sagittal plane projection, the profiles of the medial and lateral femoral condyles are different, the medial side being slightly larger, which may account for the small amount of axial rotation observed during joint flexion (Nordin and Frankel, 1980). Nevertheless, in a two-dimensional model an average condylar profile must be used to approximate the projection of both the medial and lateral articulating surfaces. Rehder (1983) reported excellent fits to each of the lateral and medial condylar profiles using mathematical spirals. Circular arcs and ellipses have also served to represent the condylar profiles (Mensch and Amstutz, 1975; Wongchaisuwat *et al.*, 1984 respectively).

An averaged outline of the tibial plateau must also be modeled. It is reported to slope posteriorly at a slight angle below a line perpendicular to the tibial axis. This slope averages 7.2° and 9.2° for women and men respectively (Nisell *et al.*, 1986).

Tibiofemoral joint model

Elliptical curves were used to represent the distal femur in the sagittal plane. Errors introduced by this approximation (< 1 mm) are smaller than the maximum radial error of ± 1.6 mm generated by averaging the profiles of the medial and lateral condyles. Separate curves were used to represent the average condylar profile and the median groove forming the patellar surface. The dimensions of these curves were obtained from measurements of a small (height 4 ft 10 in.) male skeleton in the laboratory and scaled to approximately those of an average male. The average anterior–posterior dimension of the medial and lateral condyles, shown to provide good correlation with many knee parameters (Mensch and Amstutz, 1975), was used to determine the scaling factor. Scaled parameters of the model were found to agree with the normative data of Mensch and Amstutz (1975) to within 10%, and are listed in Table 1.

Table 1. Parameter values describing the model

Description	Size (cm)
Patellar thickness, t	1.63
Patellar length, l_p	3.94
Patellar ligament length, l_{pl}	6.52
Semimajor radii of femoral articulating surfaces:	
femoral condyles	3.54, 2.18
median anterior groove*	2.86, 1.90

*Note: center of ellipse describing the patellofemoral articulating surface is displaced proximally 0.82 cm from the center of the ellipse describing the averaged femoral condyle.

A straight line sloping 8° below the normal to the tibial axis was used to represent the weight-bearing surface of the tibial plateau. The menisci were not modeled, as they do not seem to significantly alter even forced anterior–posterior translations or the accompanying tibial rotation in the healthy knee (Levy *et al.*, 1982).

Using trial and error, we interpolated ICR locations with respect to the tibia at each degree of flexion (Fig. 1B) using piecewise cubic splines fit to 4–10 control points spaced at equal increments of flexion. The coordinates of the control points were adjusted until a set of constraints, here referred to as the contact criteria, were qualitatively satisfied. First, the tibia and femur had to remain in contact (within 0.5 mm) throughout the joint motion. Second, the femur could not roll or slip off either end of the tibial plateau. Third, the movement of the *TF* contact point should travel a path along the tibial surface with knee flexion similar to that reported by Nisell *et al.* (1986) (Fig. 2). We also assumed that the tibiofemoral joint purely rolled in the fully extended position, and slipped increasingly with joint flexion so that the backward movement of the tibiofemoral contact point decreased as the joint was flexed. Figure 1A shows the effect of prescribing the ICR pathway inappropriately. Fig. 1B shows our pathway.

Patellofemoral joint

The patella should be modeled both as a spacer and as a lever. To accomplish this, we assumed in our two-dimensional model that a single-point contact exists between the femur and a rectangular patella having a patellar axis at an approximate average distance of 't' from the articulating surface (Fig. 3). Since the contact area on the patella occurs in a narrow band only between 0 and 90 degrees of flexion (Hungerford and Barry, 1979), our model is limited to this range. Xerograms of a male (6 ft 1 in.) subject exhibiting good soft-tissue definition and correlating well with the model and normative data (Mensch and Amstutz, 1975), when scaled, were used to estimate the patellar dimensions as well as patellar ligament length. With this representation, the *PF* contact point can then be found given the orientation of the patellar axis.

All bodies are considered rigid and inextensible. In particular, if the patellar ligament is treated as having constant length l_{pl}, then its intersection with the patellar axis must lie on the circle of radius l_{pl} (Fig. 4) circumscribed about the ligament's insertion on the tibia. These simplifications allow the geometrical

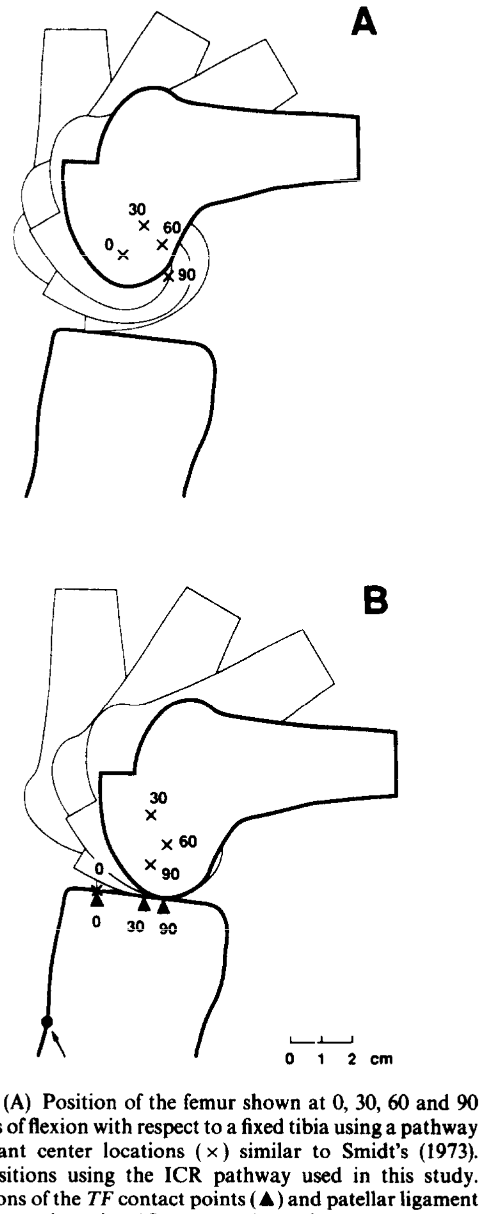

Fig. 1. (A) Position of the femur shown at 0, 30, 60 and 90 degrees of flexion with respect to a fixed tibia using a pathway of instant center locations (×) similar to Smidt's (1973). (B) Positions using the ICR pathway used in this study. Locations of the *TF* contact points (▲) and patellar ligament insertion (●, at arrow) are shown.

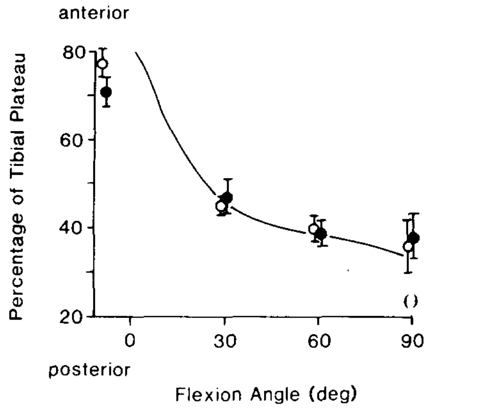

Fig. 2. Location of the Model"s *TF* contact point (solid line) across the tibial plateau with joint flexion. 100% represents the anterior border of the plateau. Shown are the data for men (●) and women (○) reported by Nisell *et al.* (1986).

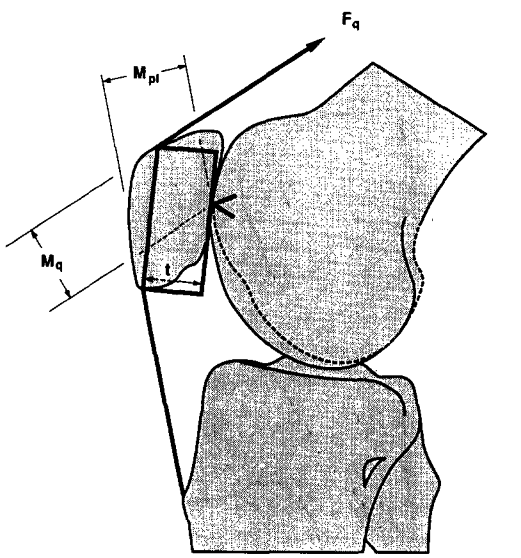

Fig. 3. Patellar model to fit to a xerogram tracing (shaded region), showing moment arms of quadriceps tendon (M_q) and patellar ligament (M_{pl}) about the *PF* contact point (<). Dashed line profiles the lateral condyle.

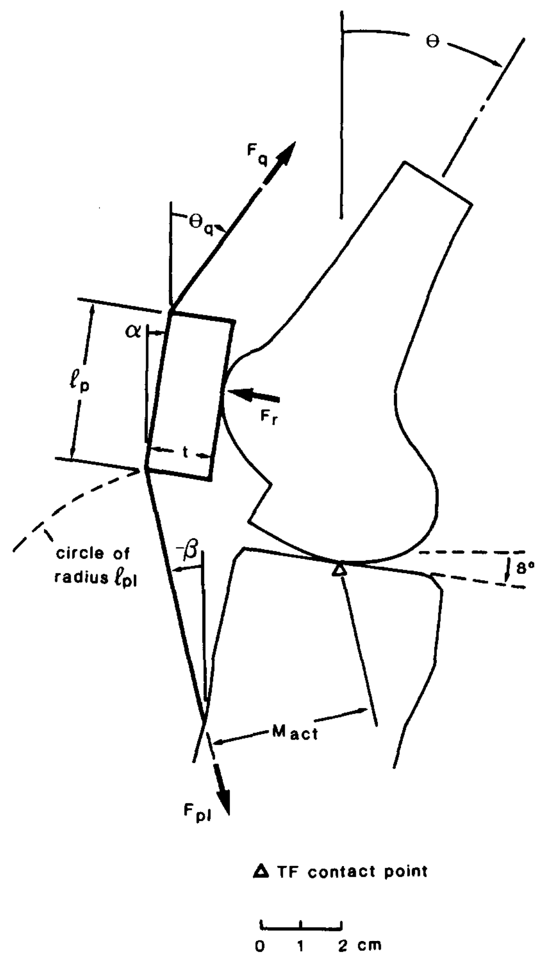

Fig. 4. Parameters and angular definitions used in the two-dimensional model.

arrangement of the patellofemoral joint model (α and β) to be calculated as functions of the flexion angle (θ) alone. Since the direction of the applied quadriceps force (θ_q) and the orientation of the femur with respect to the fixed tibia are also prescribed functions of flexion, θ remains the single independent parameter describing the joint.

Numerical procedure

In increments of 1° from full extension to 90° of flexion, the inclination of the patella was found using Newton–Raphson iteration to solve the following equations governing static equilibria of the patellar mechanism. We ignored friction, as the coefficient of friction in synovial joints has been found to be extremely small (Radin and Paul, 1972). The balance of forces is given by

$$\begin{bmatrix} \cos\alpha & \sin\beta \\ -\sin\alpha & \cos\beta \end{bmatrix} \begin{bmatrix} F_r \\ F_{pl} \end{bmatrix} = \begin{bmatrix} \sin\theta_q \\ \cos\theta_q \end{bmatrix} F_q \qquad (1)$$

where F_q represents the applied quadriceps force of arbitrary magnitude (since all bodies are inextensible). F_r and F_{pl} are the unknown patellofemoral compression force and patellar ligament tension. F_{pl} must also satisfy moment equilibria

$$F_{pl}M_{pl} = F_q M_q \qquad (2)$$

where M_{pl} and M_q are the moment arms of the patellar ligament and quadriceps tendon about the *PF* contact point (Fig. 3). Since β can be found easily from α and M_{pl} and M_q are also known functions of α, the system of three equations can be solved for the three unknowns α, F_{pl} and F_r. With an initial guess of α, typically 4–5 iterations were required for F_{pl} to converge to within 1.5%.

The 'actual' moment arm (M_{act}) of the joint was calculated as the perpendicular distance from the central axis of the patellar ligament to the *TF* contact point. Multiplying the actual moment arm by the ratio of transmitted to applied force, we then found the 'effective' moment arm (M_{eff})

$$M_{eff} = \frac{F_{pl} M_{act}}{F_q}. \tag{3}$$

Figure 5A shows the initial and final configurations of the knee using quadriceps orientations reported by van Eijden *et al.* (1985). Figure 5B shows the configurations if the quadriceps force is directed parallel to the femoral axis, as assumed in other studies. Note in the latter case that the axis of the quadriceps tendon contacts the femur at very large flexion angles (arrow). When such contact occurred (at flexions greater than 80°), we assumed that the quadriceps tendon wrapped around the femoral profile without friction. Thus the direction, but not the magnitude, of the quadriceps force was altered. Contact forces were neglected at the tendon–femur interface.

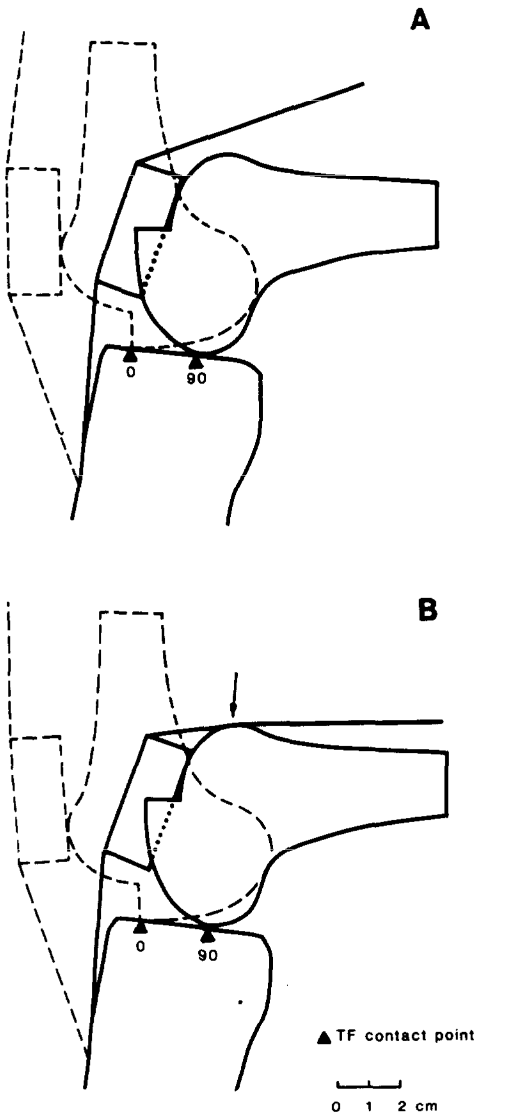

Fig. 5. Joint configuration shown at 0 (dashed) and at 90° (solid) for two different alignments of the quadriceps tendon. (A) Direction of the quadriceps force is prescribed using anatomical data reported by Van Eijden *et al.* (1985). (B) Quadriceps force is assumed colinear with the femoral axis throughout the flexion range. Note tendon–femur contact (arrow).

RESULTS

Our simple, single degree-of-freedom knee joint model matches well the observed, sagittal-plane behavior of the knee. This conclusion is based on comparison of the location of the *TF* and *PF* contact points, the angular orientations of the patella and patellar ligament and the calculated forces and moment arms with reported experimental data at different joint angles.

For instance, Figs 6B and C show good agreement between both the orientation of the patella's axis (α)

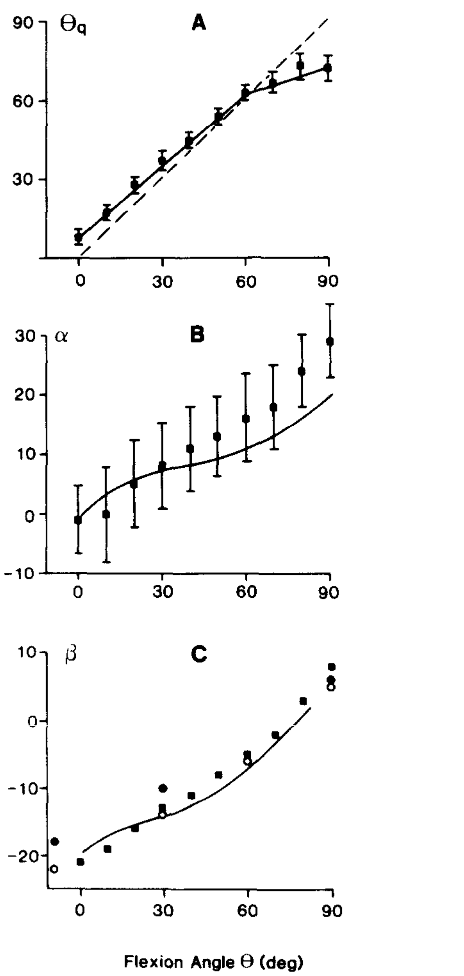

Fig. 6. (A) Quadriceps force orientation in our model (solid line) was prescribed to match data of van Eijden *et al.* (1985) (■). Dashed line shows femoral axis orientation. Orientations of the patellar axis (B, solid line) and the patellar ligament (C, solid line) calculated from our model are in agreement with data adapted from Van Eijden *et al.* (1985) (■) and Nisell (1985) (●, men and ○, women shown in C only). Error bars are omitted for clarity in (C).

and ligament (β) computed from our model and data taken from X-rays reported by others, assuming that the orientation of the quadriceps tendon with respect to the femur varies with flexion (Fig. 6A). This supports the premise that the femur is moving properly in relation to the tibia and further implies that all of the relative dimensions are approximately correct. Thus, the planar model represents the sagittal-plane projection of the three-dimensional joint quite well.

We found that the forces in the patellar ligament and at the *PF* contact point (Figs 7A and B) calculated from our two-dimensional model compared well with those computed from a more complex two-dimensional model of the patellofemoral joint (van Eijden *et al.*, 1986). The patellofemoral joint of our single degree-of-freedom knee model thus essentially replicates the same mechanical processes as the more complex model. The results of Nisell (1985) are also shown in Fig. 7 for comparison.

Mechanics of the quadriceps effective moment arm

Since we found that the qualitative variation of our computed actual and effective moment arms with flexion agrees with experimental data (Fig. 8), our knee model can be used to understand the behavior of the recently measured curves of effective moment arm vs flexion angle. The reason for the steep rise at small flexions is due primarily to the pronounced posterior movement of the *TF* contact point away from the axis of the patellar ligament (Figs 1B and 2), which increases the actual, and hence the effective, moment arm. Since the *PF* contact point, which acts as the fulcrum of the patellar lever, is near the distal end of the patella when the knee is extended, the potential mechanical force advantage of the lever is greatest near full extension. However, the levering effect is minimal with the joint extended because the lines of action of the quadriceps and patellar ligament forces are closely aligned with the patellar axis. Thus, the patellar ligament to quadriceps force ratio reaches a maximum of only about 1.1 (at 0°, Fig. 7A). The actual and effective moment arms at low flexion angles are therefore very similar.

For flexions beyond 30°, the incremental movement of the *TF* contact point per degree of flexion decreases as the amount of slippage at the tibiofemoral interface increases (Figs 1B and 2). Concurrently, the patellar ligament swings toward the *TF* contact point, as the patella moves posteriorly into the deepening median groove of the femur. The ensuing rotation of the patellar ligament about its insertion acts to reduce the actual moment arm (Fig. 8A).

Also, with increasing knee flexion, the *PF* contact point migrates proximally across the posterior surface of the patella, thereby decreasing the mechanical force advantage of the patellar lever. This is consistent with

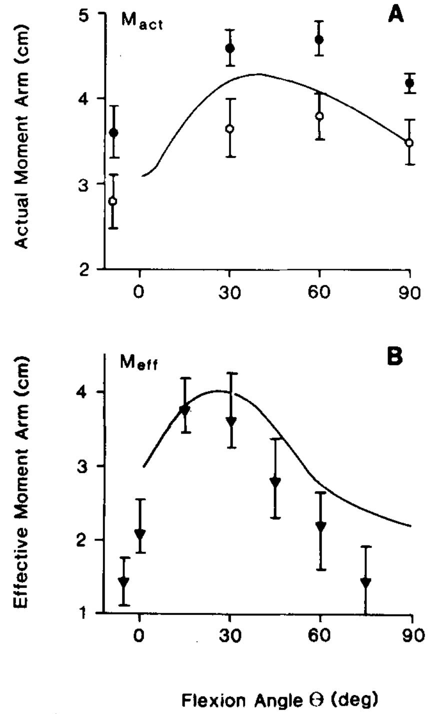

Fig. 8. (A) Comparison of the actual moment arm calculated from our model (solid line) with data presented by Nisell (1985) for men (●) and women (○). (B) Comparison of the effective moment arm calculated from our model (solid line) with the experimental data of Grood *et al.* (1984). Error bars in (B) represent the range of values reported while the symbols (▼) reflect the average.

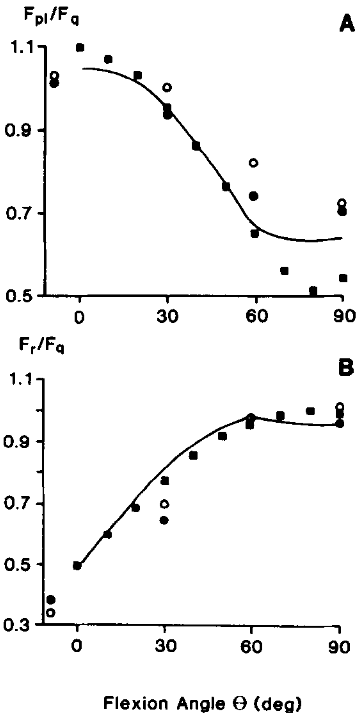

Fig. 7. Ratios of (A) patellar ligament force and (B) *PF* contact force to quadriceps tendon force calculated from our model (solid lines) compared with results from Nisell (1985) (●, men and ○, women) and van Eijden *et al.* (1986, ■). Error bars are omitted for clarity.

experimental observations (Goodfellow *et al.*, 1976; Hungerford and Barry, 1979) and the fact that the patella acts to *reduce* the force transmitted from the quadriceps tendon to the patellar ligament (Fig. 7A). The combined effects of a decreasing actual moment arm and a declining transmitted force with flexion results in the steep fall of the effective moment arm curve.

Our results are sensitive to the direction of the applied quadriceps force. For instance, notice the difference when the quadriceps force was applied parallel to the femoral axis (dashed line in Fig. 9A) instead of at the orientations measured by van Eijden *et al.* (1985) (solid line, Fig. 9A). Although the actual moment arm (not shown) is not significantly affected by the direction of F_q, the patellar ligament force is.

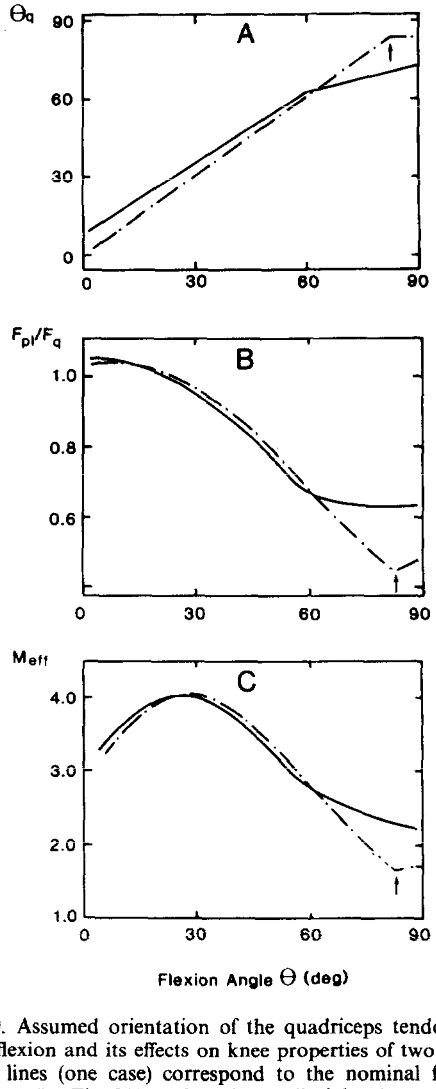

Fig. 9. Assumed orientation of the quadriceps tendon (A) with flexion and its effects on knee properties of two cases. Solid lines (one case) correspond to the nominal flexion sequence (i.e. Fig. 5A) used to obtain all of the above results, where the quadriceps force is not necessarily parallel to the femoral axis. Dashed-dotted lines (second case) correspond to the case where quadriceps force is indeed assumed parallel to the femoral axis (see Fig. 5B). Arrows indicate changes in slope resulting from contact between the quadriceps tendon and the femur.

Between 60 and 90°, F_{pl} remains relatively constant with flexion when the orientation of the quadriceps force is not assumed to be colinear with the femur, but a substantial reduction in force is apparent when colinearity is assumed (Fig. 9B). The absence of a reduction in force in the former case is due to the fact that the quadriceps tendon and patellar ligament remain more aligned with flexion. The effective moment arm curves (Fig. 9C) exhibit similar differences, as M_{eff} is directly proportional to F_{pl} by equation 3.

When the quadriceps orientation is maintained parallel to the femoral axis, the tendon axis contacts the femur at approximately 82° (arrows, Fig. 9), altering the direction of F_q. Again, the results demonstrate sensitivity to θ_q.

Effect of patellar size and ligament length on the effective moment arm and other knee properties

For greater understanding of the knee, the dimensions of the patella and the length of the patellar ligament were varied.

Patellar thickness. Sensitivity of knee properties to a thicker patella (by 20%) is shown in the leftmost column of Fig. 10. Observe that the actual moment arm is increased over the whole flexion range (*E*), showing the anticipated result that a thick patella displaces the patellar ligament further from the tibiofemoral contact point. However, the effective moment arm is only sensitive to patellar thickness at flexions less than 35° (*F*). The reason is that at high flexions, the force transmitted from the quadriceps tendon to the patellar ligament is low, even when the patella is thick (*C*). The sensitivity of M_{eff} to changes in M_{act} is reduced, as it is directly affected by this force transmission, i.e. from equation (3).

$$\frac{\partial M_{eff}}{\partial M_{act}} = \frac{F_{pl}}{F_q}. \tag{4}$$

Patellar length. The effect of changes in patellar length on knee properties are shown in Fig. 10 (center column). Since the length of the patellar ligament was held constant, an increase in l_p means an increase in length only on the proximal side of the *PF* contact point. This increases the mechanical force advantage of the lever by lengthening M_q about the *PF* contact point, causing greater deflections of the patellar axis and ligament (*G*, *H*). The greater patellar ligament deflection leads to a larger actual moment arm (*K*). Since the quadriceps-force moment at the *PF* contact increases from the longer quadriceps moment arm, the patellar ligament force (*I*) also increases to counter this quadriceps moment. Thus, quadriceps force transmission to the patellar ligament is enhanced. The higher ligament force, besides causing a higher *PF* contact force (*J*), acts together with the larger actual moment arm (see equation 3) to cause a substantial increase in the effective moment arm (*L*).

Patellar ligament length. A longer patellar ligament affects the effective moment arm more than a longer

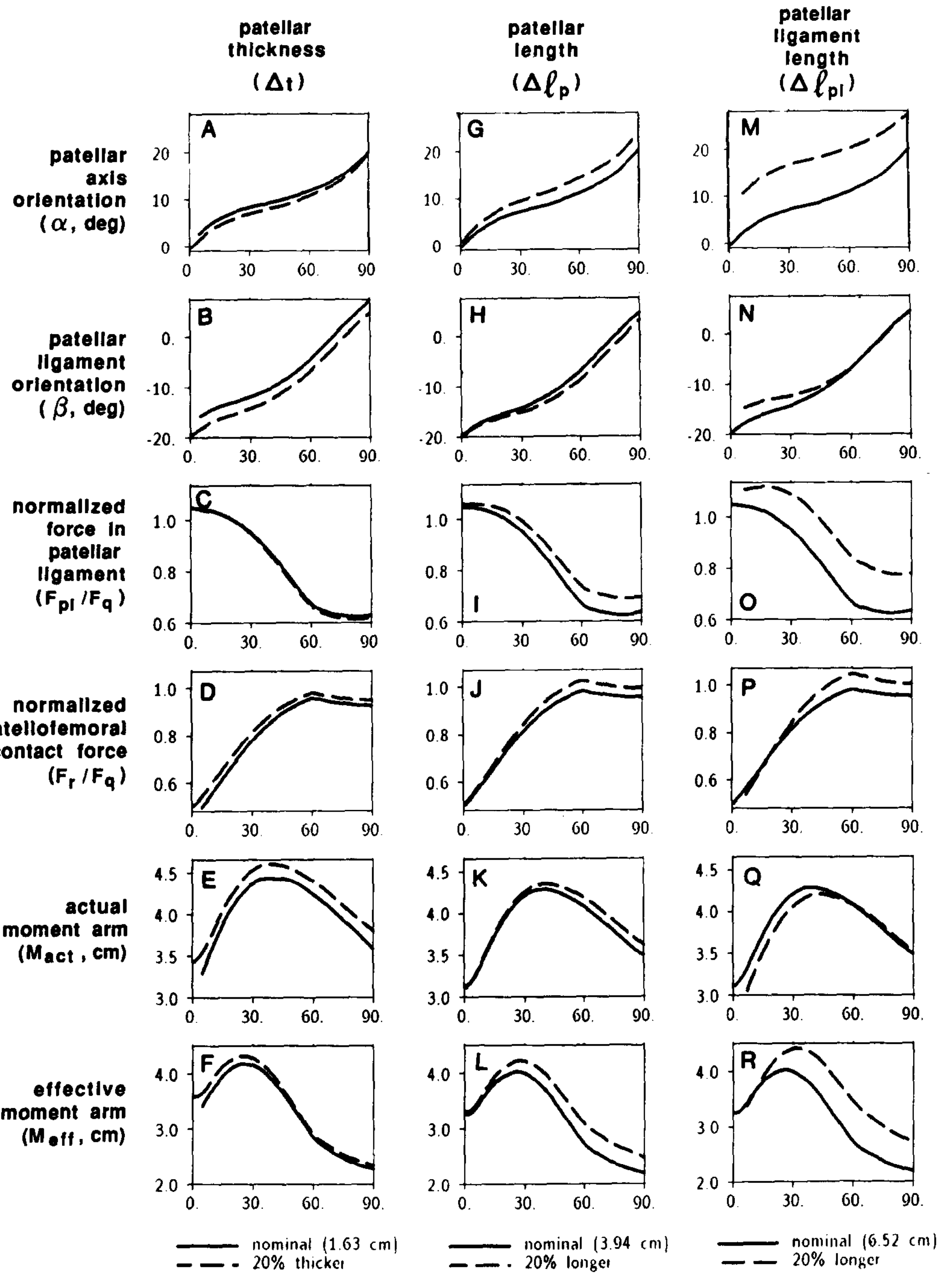

Fig. 10. The effects of increasing patellar thickness (*A–F*), patellar length (*G–L*), and patellar ligament length (*M–R*) on knee properties calculated from the two-dimensional knee model. Curves corresponding to nominal parameter values are shown as solid lines. Dashed lines indicate values 20% larger.

patella (Fig. 10, compare *R* with *L*). This enormous effect on the effective moment arm by a longer ligament is due not only to an increase in M_q but also to a decrease in M_{pl}, causing quadriceps force transmission to the patellar ligament to be very much accentuated (see equation 2 and Fig. 10, *O*).

DISCUSSION

Our results confirm that the levering action of the patella is at least as important to joint extension as its function as a spacer. While thicker patellae do indeed increase the effective extensor moment arm, the relatively modest gains are confined to flexions less than about 35°. A thick patella does not affect the effective moment arm at high flexions because it does not change the patellar levering properties, which dominate the effective moment arm at high flexions. The reason that a thick patella increases M_{eff} at low flexions is because it increases the actual moment arm, which dominates the effective moment arm near full extension. In contrast, the lengths of the patella and its ligament substantially affect the levering action of the patella and, therefore, the effective moment arm at all flexions above 15°. Thus a longer patella, and especially a longer ligament, enhance the transmission of quadriceps force to the patellar ligament (Fig. 10, *I* and *O*) to increase the effective moment arm (Fig. 10, *L* and *R*).

Because patellar leveraging is significant, the effective moment arm depends on the directions of the quadriceps and patellar ligament forces as well as on their magnitudes. Indeed, we found the orientation of the quadriceps force to significantly affect M_{eff}, but only at high flexions (Fig. 9C). However, Grood *et al.* (1984) report that the effective moment arm is only slightly affected by the orientation of the quadriceps force. We are therefore in agreement except in the case of high flexions. Unfortunately, we have no explanation for these differences.

Of course, some differences with experimental results are expected. For example, we assume that the motions and orientations of the tibia, femur, patellar axis, patellar ligament and quadriceps tendon are coplanar throughout the flexion range. This is not a good assumption near full extension due to the 'screw-home' mechanism of the joint. Yet, the small axial rotations of the tibia relative to the femur are not expected to significantly alter the basic mechanism illustrated with this model, as the errors in projecting these motions onto the sagittal plane are expected to vary with the cosine of the axial rotation angle.

Perhaps of greater significance, the flat and elliptical surfaces used to represent the patella, femur and tibia are only approximations to the plane projections of the three-dimensional structures. Changes from the assumed straight line representing the profile of the tibial plateau would require adjustments to the ICR pathway. Based on our studies we do not expect, however, such alterations to significantly change the relative movement of the tibiofemoral contact point with joint flexion. Therefore, the trends seen with this model should predict actual ones.

The rectangular patella assumed here is also an approximation to the anatomical one. Our xerograms indicate that in the sagittal plane, the rectangular patella is about 2 mm thinner at its proximal end and 2 mm thicker at its (distal) contact point in full extension. Some indication of the differences to be expected had we used a more anatomical-like patella in our model can be seen by interpolating between the curves generated with different patellar thicknesses. At small flexions, the patella may be considered a bit thinner than the thickness used in our model. Since patellar thickness primarily affects the effective moment arm only at small flexion angles, the ascending region of the effective moment arm curve would be expected to show a slight decrease in magnitude (Fig. 10, left column).

The fact that the patellar ligament was treated as an inextensible body will also influence the results. Some idea of the effects of ligament stretch may be obtained by interpolating between the curves generated with different patellar ligament lengths (Fig. 10, rightmost column). In most normal activities, however, the axial force in the patellar ligament is expected to remain relatively small, so that the stretch in the ligament can be neglected. For instance, based on patellar ligament properties reported by Butler *et al.* (1984), we estimate 7% strain in the ligament during maximum isometric contractions. In less strenuous activities such as walking, our estimate is less than 1%.

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