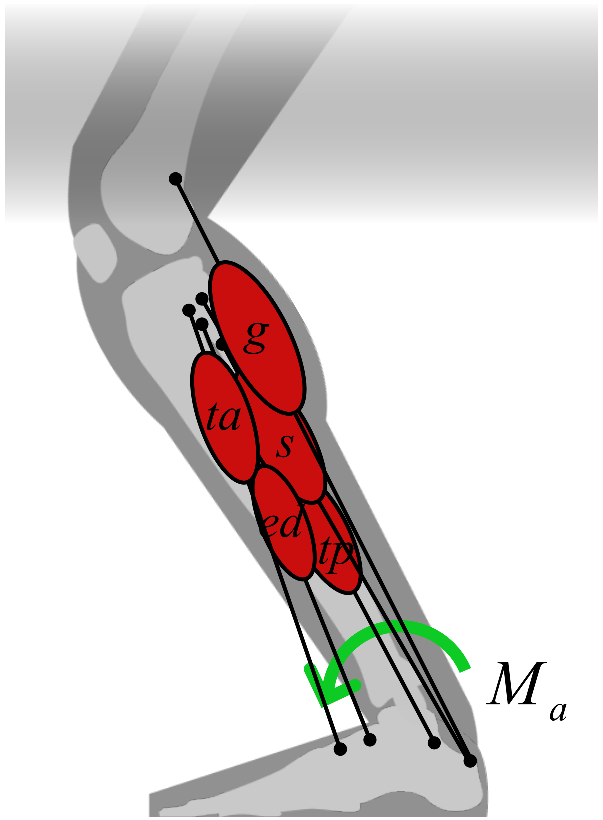
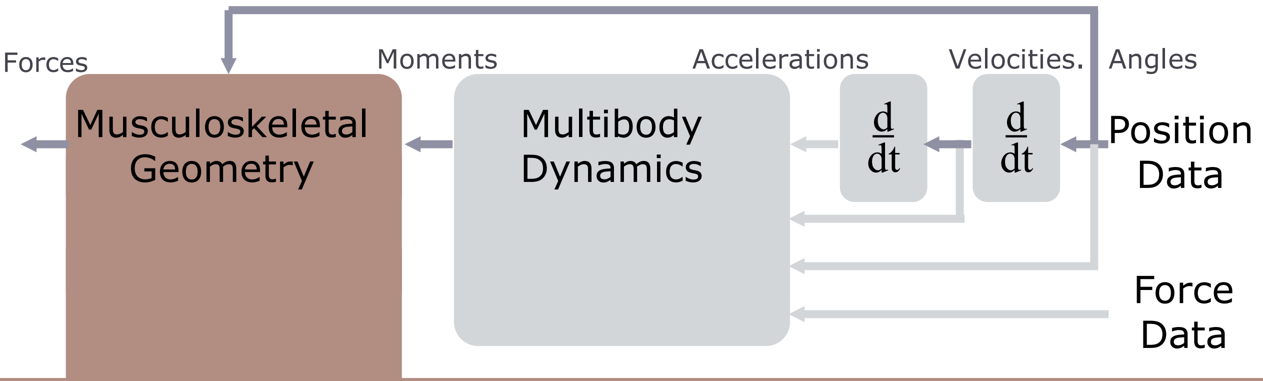


Static Optimization

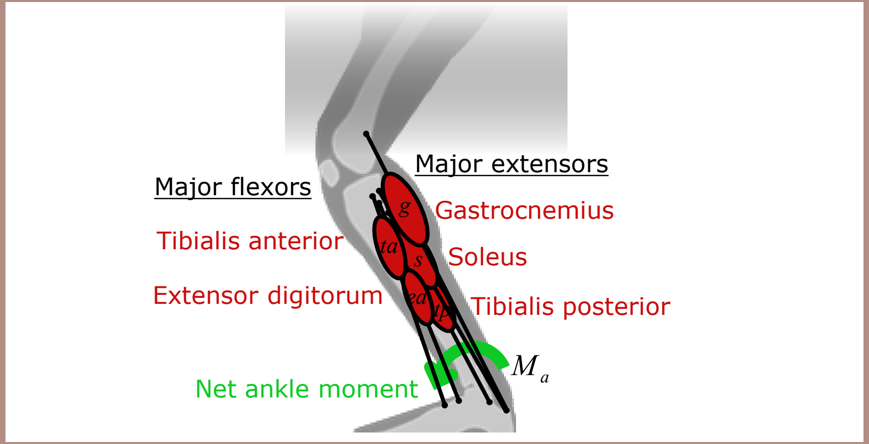
The Inverse Problem



Static Optimization

Inverse Dynamics

Inverse Kinematics



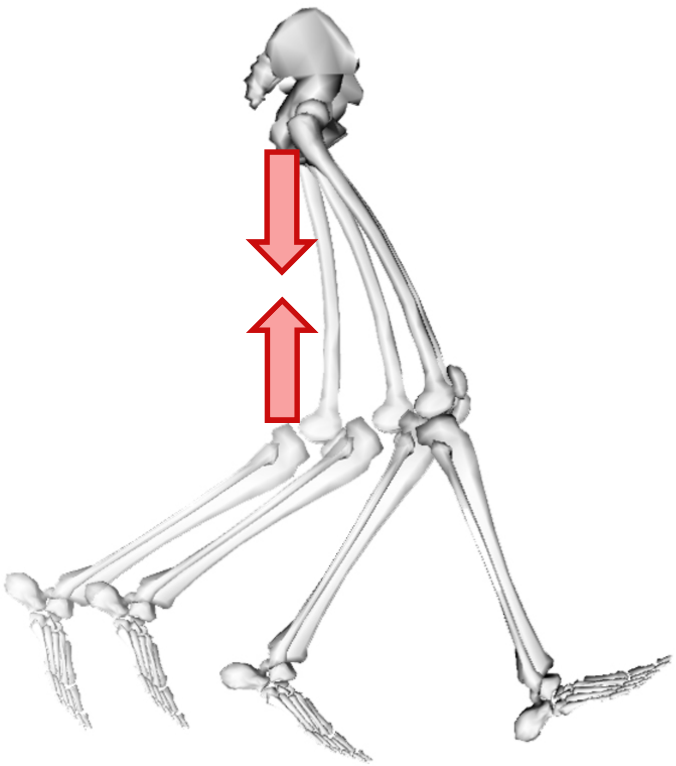
- Use musculoskeletal geometry and assumptions about force distribution to estimate individual muscle forces

Key Concepts

• **Kinematics**	coordinates and their velocities and accelerations
• **Kinetics**	muscle forces
• **Muscle physiology**	muscle activation-contraction dynamics and force-length-velocity relations
• **Dynamics**	equations of motion
• **Musculoskeletal geometry**	muscle moment arm
• **Optimization**	the "distribution" problem

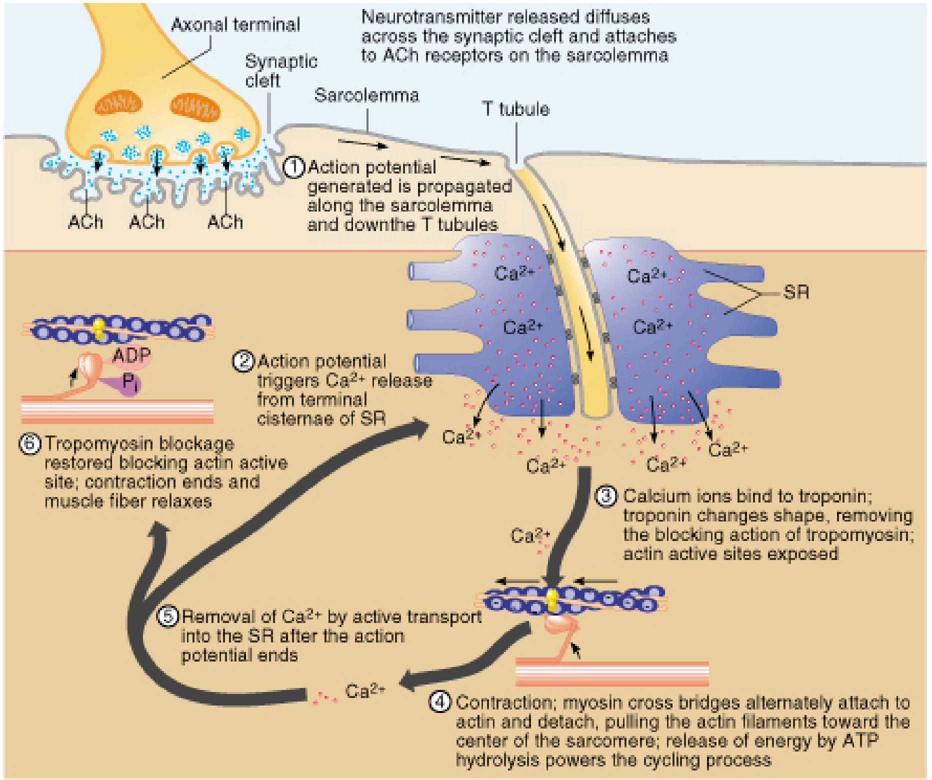
Kinetics: Muscle Forces

- Kinetics
  - Muscle forces cause the model to accelerate
    - Muscle force
      - Applied between origin and insertion points



Muscle Physiology: Muscle Activation-Contraction and Force-Length-Velocity Relations

- Muscle activation-contraction
  - Biochemical reaction that causes a muscle's fibers to contract which produces force



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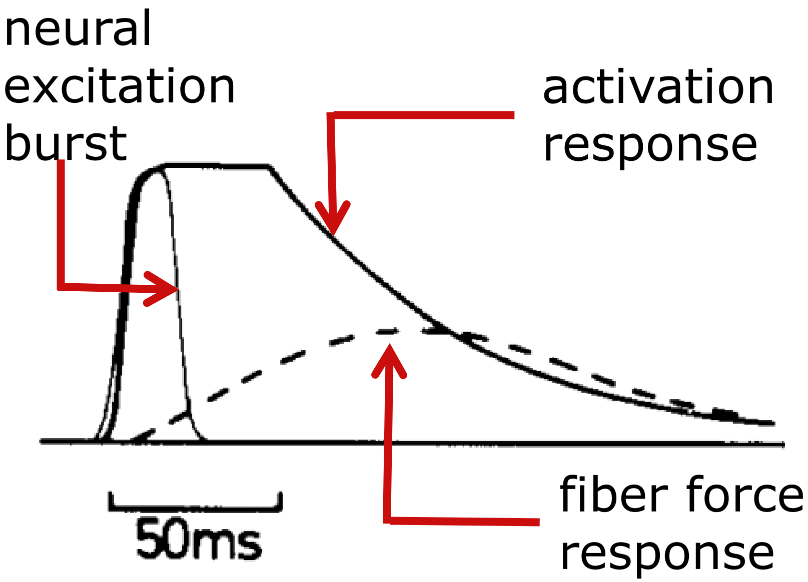
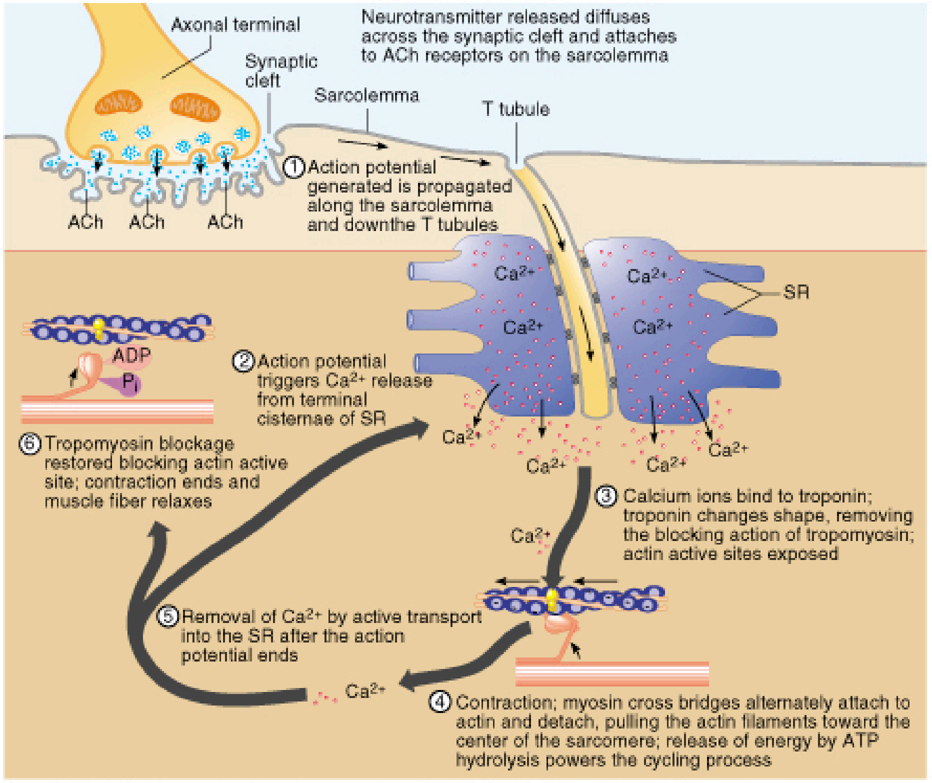
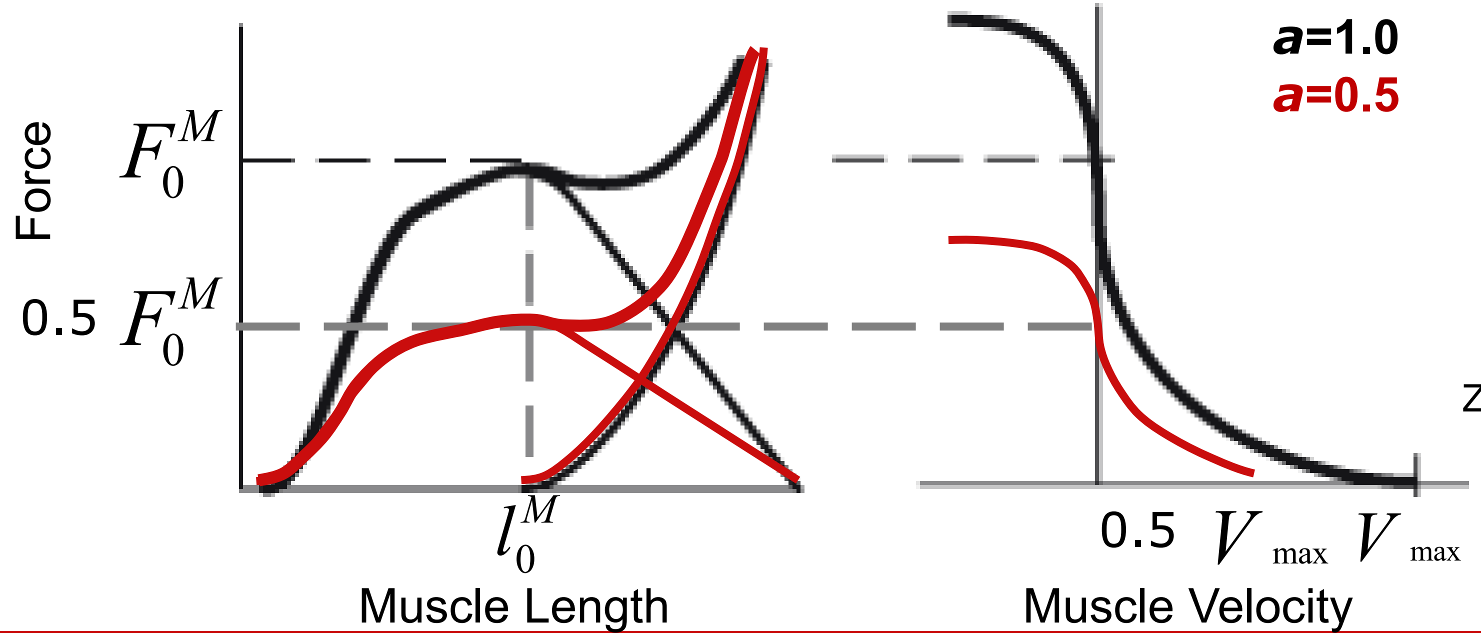




Hof, Human Movement Science, 1984

Muscle Physiology: Muscle Activation-Contraction and Force-Length-Velocity Relations

- Muscle activation-contraction
  - Biochemical reaction that causes a muscle's fibers to contract which produces force
- Muscle force-length-velocity
  - Force production diminshes for short, long, and fast fibers

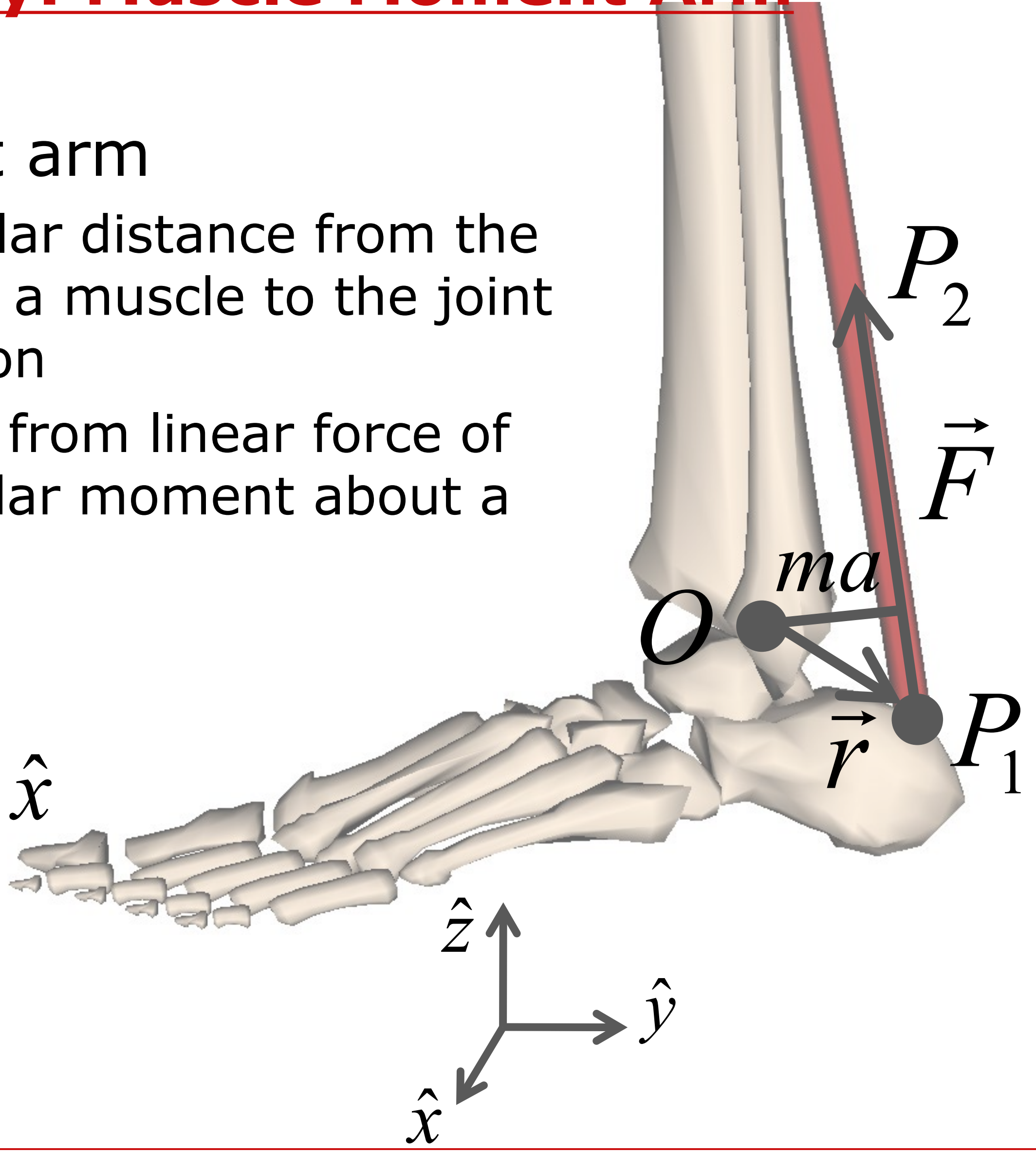


Zajac, Crit Rev Bioeng, 1989

Musculoskeletal Geometry: Muscle Moment Arm

- Muscle moment arm
  - The perpendicular distance from the line of action of a muscle to the joint center of rotation
  - Transformation from linear force of muscle to angular moment about a joint center

$$ma_x = \frac{\vec{r} \times \vec{F}}{\left|\vec{F}\right|} \bullet \hat{x}$$



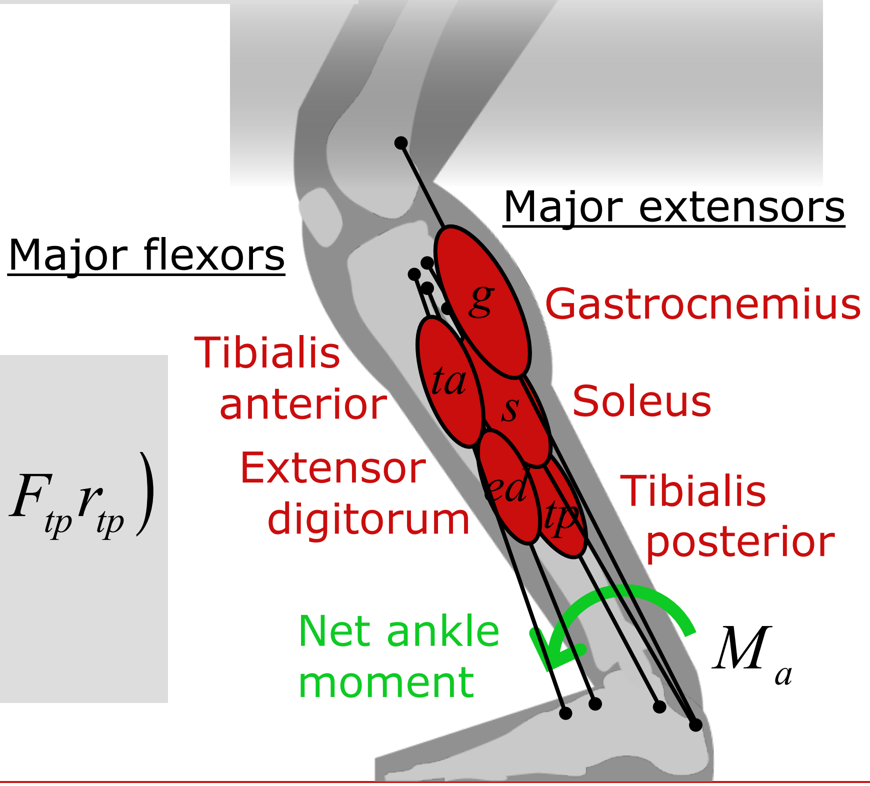
The Muscle Force Distribution Problem

$$M_j = \sum \text{muscle moments} + \sum \text{moments due to other structures}$$

$$M_j = \sum_{f=1}^{nf} F_f r_f - \sum_{e=1}^{ne} F_e r_e$$

- number of flexors: nf
- number of extensors: ne
- moment arm: r_f, r_e
- flexion moment: $\sum_{f=1}^{nf} F_f r_f$
- extension moment: $\sum_{e=1}^{ne} F_e r_e$

1 equation with $n_f + n_e$ unknowns



Ankle example

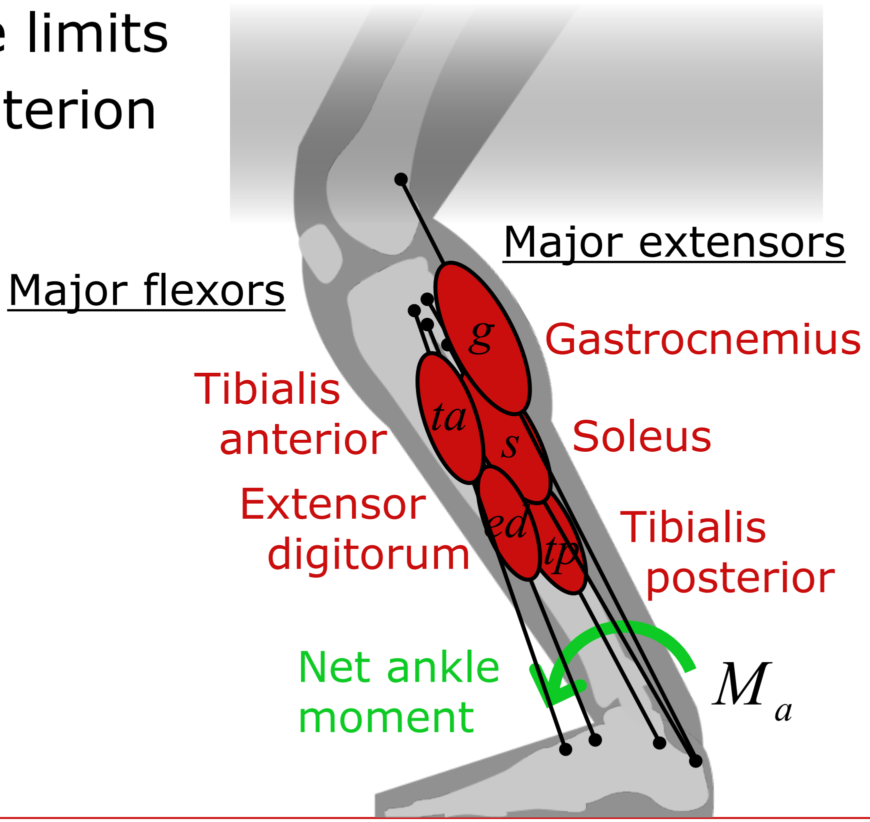
$$M_a = \left(F_{ta} r_{ta} + F_{ed} r_{ed}\right) - \left(F_g r_g + F_s r_s + F_{tp} r_{tp}\right)$$

How can we solve this?

Static Optimization

- Determines the “best” set of muscle forces that
  - Produce net joint moments at a discrete time
  - Do not violate muscle force limits
  - Optimize a performance criterion

- Performance criterion attempts to capture the goal of the neural control system
  - Minimize muscle force?
  - Minimize muscle stress?

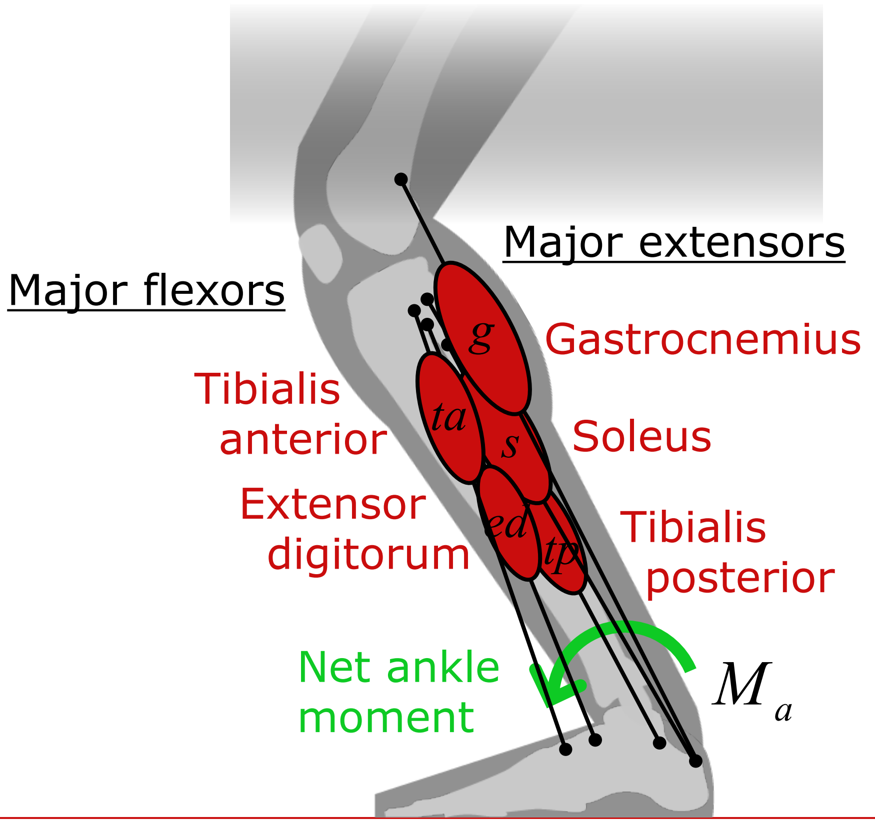


Static Optimization Formulation

minimize $f(F_m)$ Function of muscle forces

subject to $M_a(t) = [F_{ta}(t)r_{ta}(t) + F_{ed}(t)r_{ed}(t)] - [F_g(t)r_g(t) + F_s(t)r_s(t) + F_{tp}(t)r_{tp}(t)]$

$$F_{ta}(t) \leq 900\text{N}$$
$$F_{ed}(t) \leq 800\text{N}$$
$$F_g(t) \leq 1500\text{N}$$
$$F_s(t) \leq 2500\text{N}$$
$$F_{tp}(t) \leq 1500\text{N}$$



Example Performance Criteria

$$f(F_m) = \sum_{m=1}^{nm} F_m$$

Muscle force

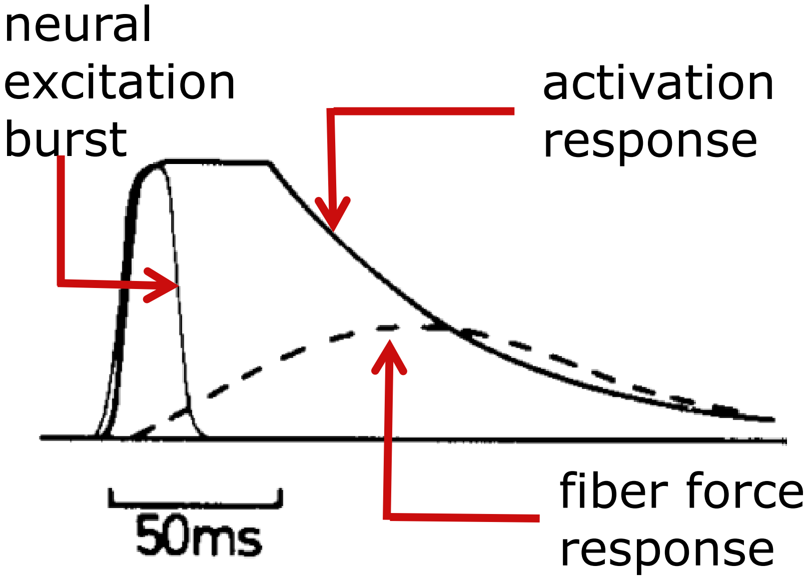
Difficult to define and validate a good criterion

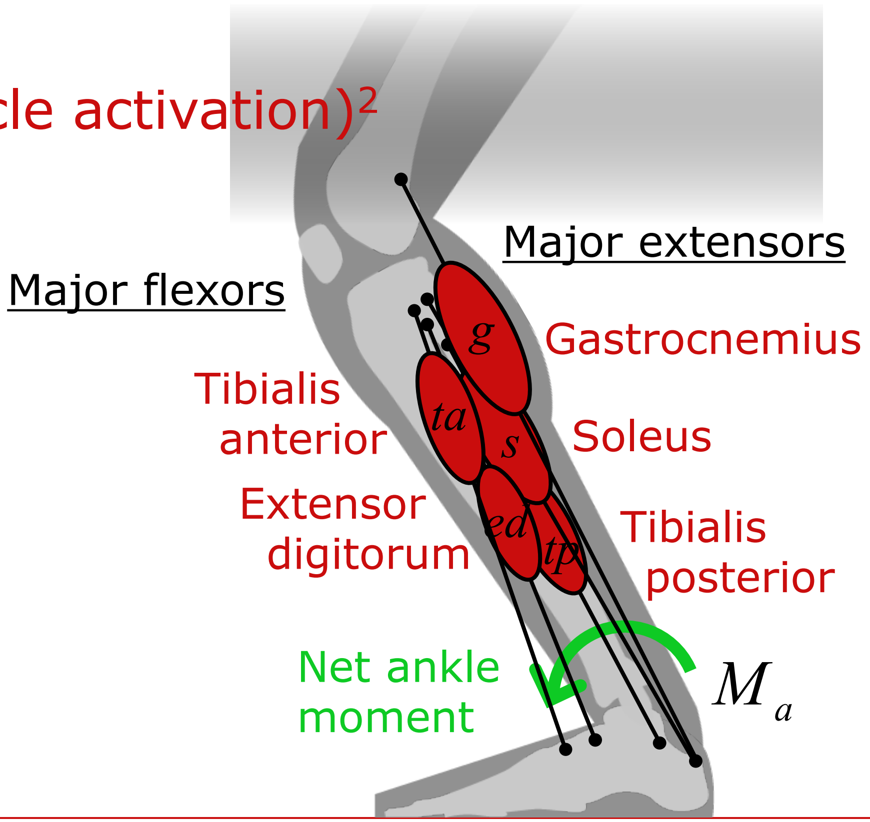
$$f(F_m) = \sum_{m=1}^{nm} \left(\frac{F_m}{PCSA_m} \right)^3$$

(Muscle stress)^3 ~ Metabolic energy

$$f(F_m) = \sum_{m=1}^{nm} \left(k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2$$

(Muscle activation)^2





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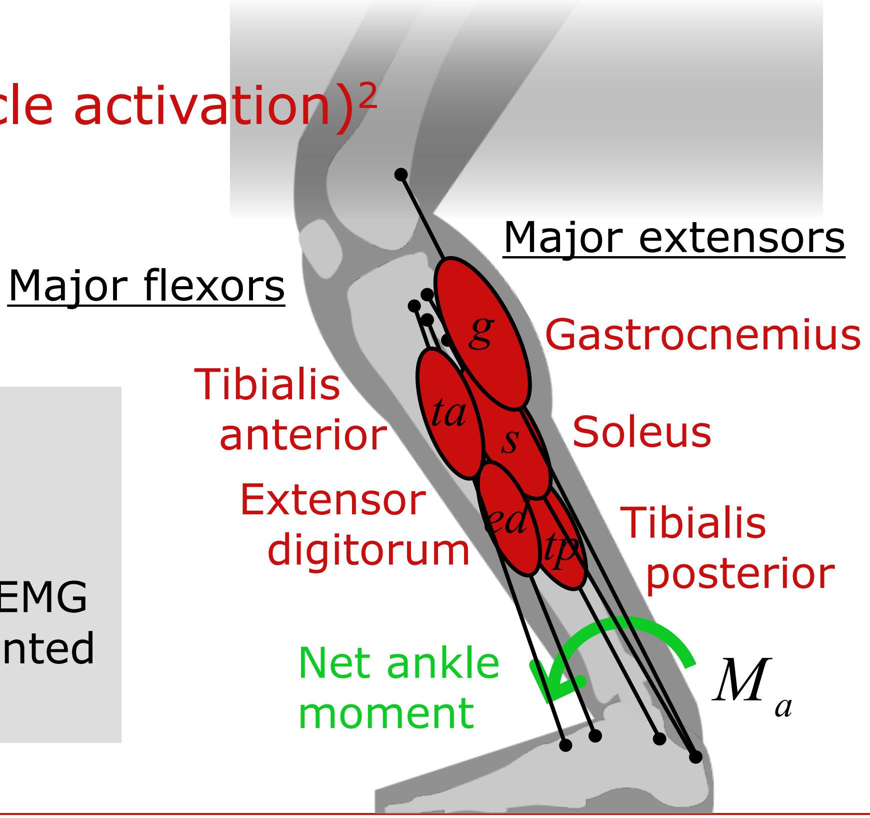

Possible validations

- Use output to drive a forward dynamic simulation
- Compare qualitatively to experimental EMG
- Compare to measured forces (instrumented implant, buckle transducer in tendon)

Example Performance Criteria

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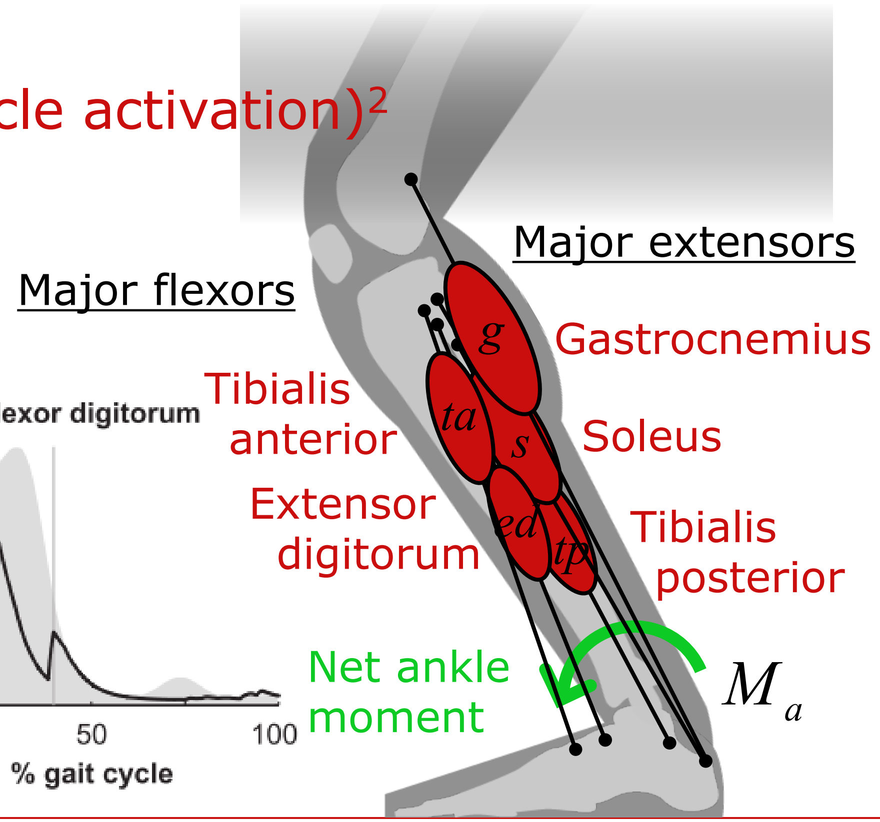
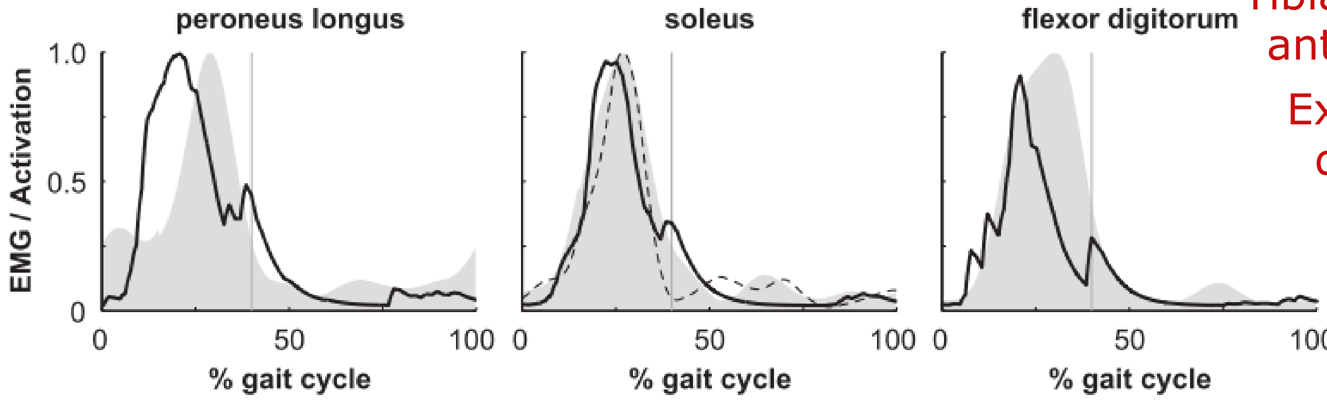




Hamner and Delp, J. Biomech., 2010

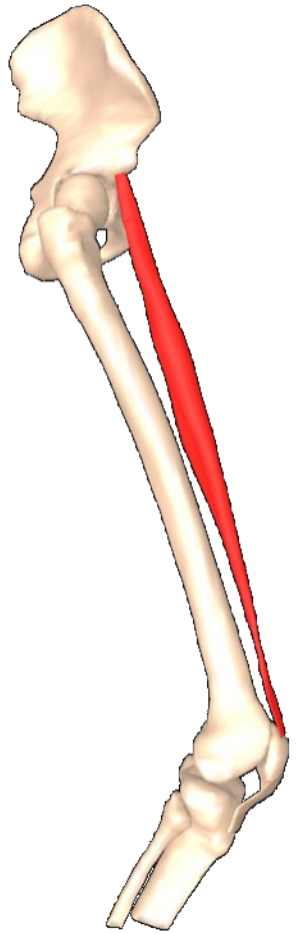
Static Optimization

TIPS & TRICKS
Inputs: Can use kinematics from IK or RRA. If using IK, need to filter kinematics
Residuals: Add residual actuators to pelvis
Reserves: Add reserve torque actuators to trouble-shoot a weak model
Minimizing residuals & reserves: Increase maximum control value (default = 1) and lower the maximum force → penalizes activity
Command Line: analyze –S setup_file.xml

Exercise

1. Given that the rectus femoris muscle has a peak isometric force of 1169 N and it is at its optimal fiber length and zero velocity, what is the force generated for an activation of 0.86?

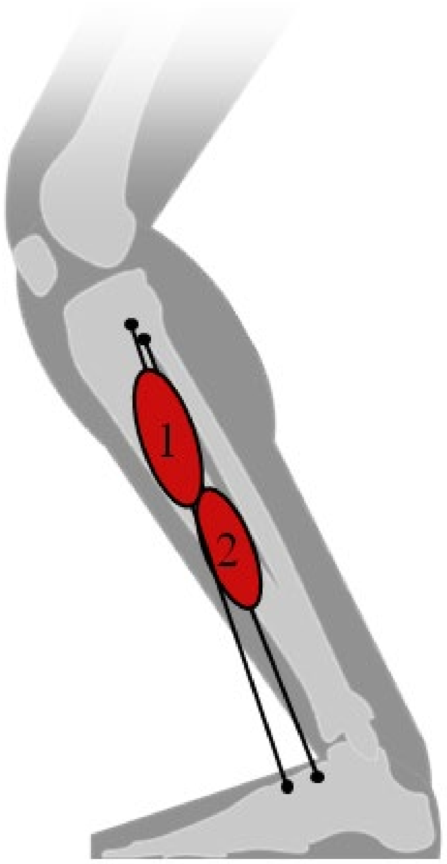
   A. 164 N
   B. 952 N
   C. 1005 N
   D. 1058 N



Exercise

2. For the model shown on the right, which muscle has the largest moment arm about the **ankle** joint?

   A. 1
   B. 2
   C. Neither (are identical)



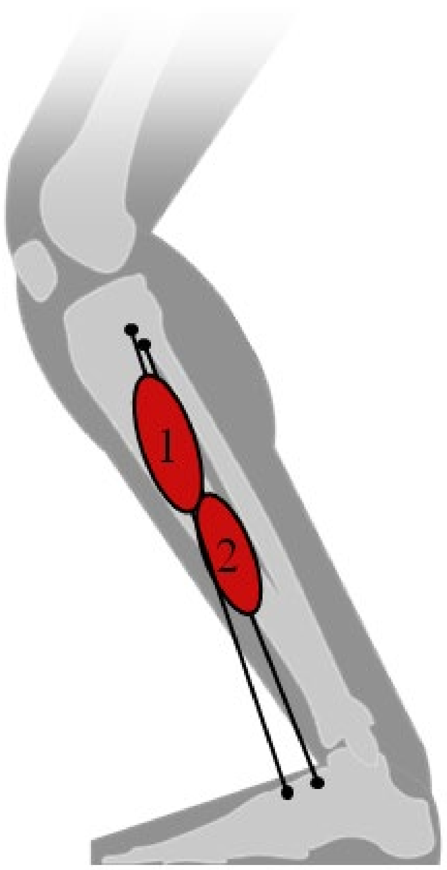
Exercise

3. For the model shown on the right, which muscle has the largest moment arm about the **knee** joint?

   A. 1

   B. 2

   C. Neither (are identical)

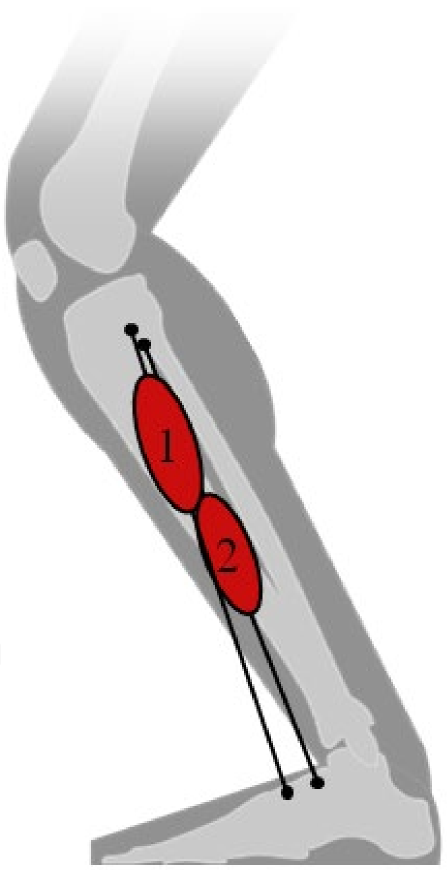


Exercise

4. For the model shown on the right, muscle 1 and 2 have the following properties

Muscle	Peak Isometric Force (N)	Moment Arm (cm)
1	905	3.6
2	512	3.0

To solve the "distribution" problem minimizing the sum of squared activations, which muscle would be activated more for a given dorsiflexion moment?



A. 1
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C. Neither (are identical)

Tracking Simulation