

## Inverse Dynamics

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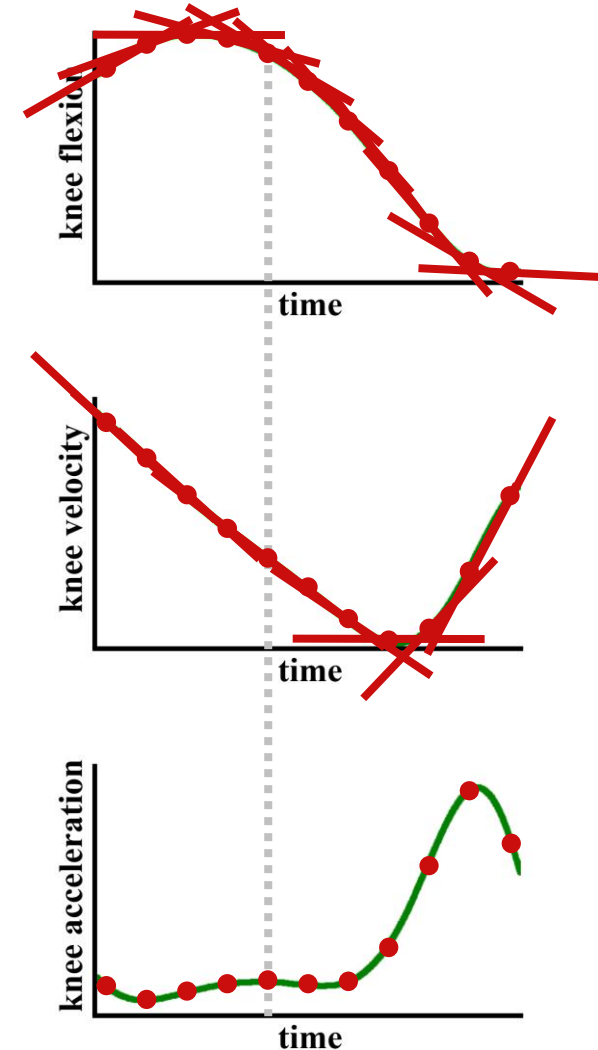
OpenSim Workshop

## **Key Concepts**

- Kinematics: coordinates and their velocities and accelerations
- Kinetics: forces and torques
- Dynamics: equations of motion

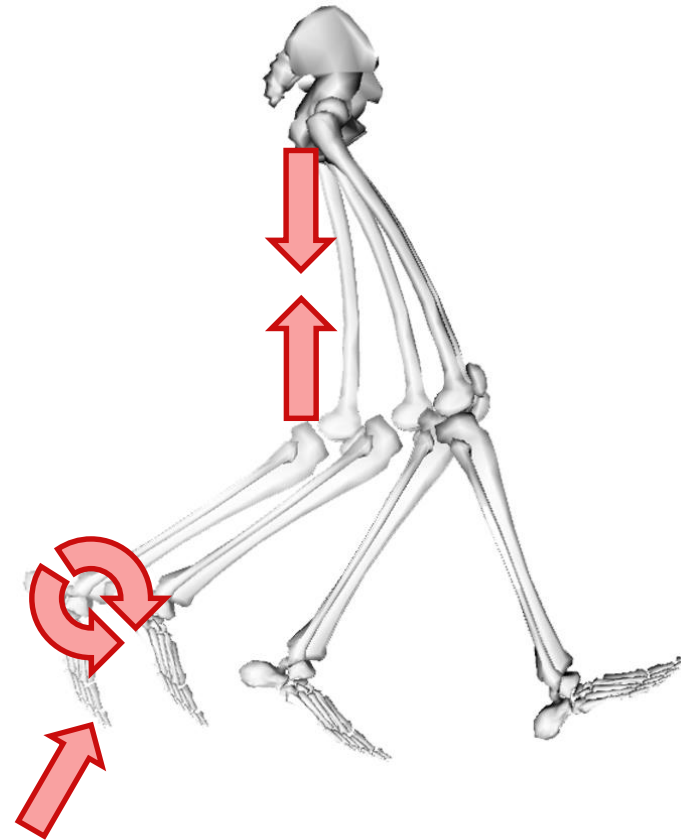
# Kinematics: Coordinates and their Velocities and Accelerations

- Coordinate
  - Joint angle or distance specifying relative orientation or location of two body segments
- Coordinate velocity
  - Derivative (rate of change) of a coordinate with respect to time
- Coordinate acceleration
  - Time derivative of a coordinate velocity with respect to time
- Kinematics
  - Set of all coordinates and their velocities and accelerations

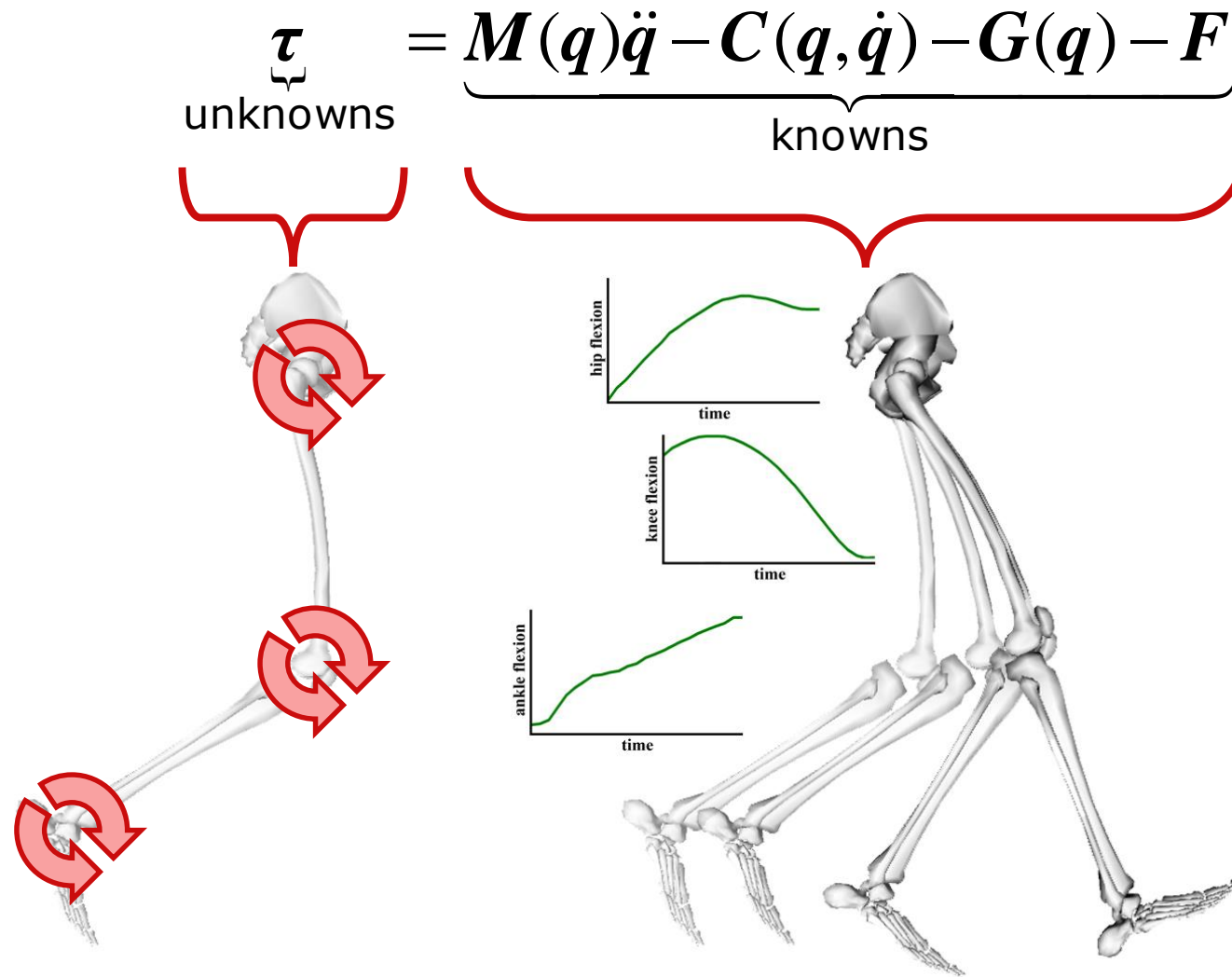


# Kinetics: Forces and Torques

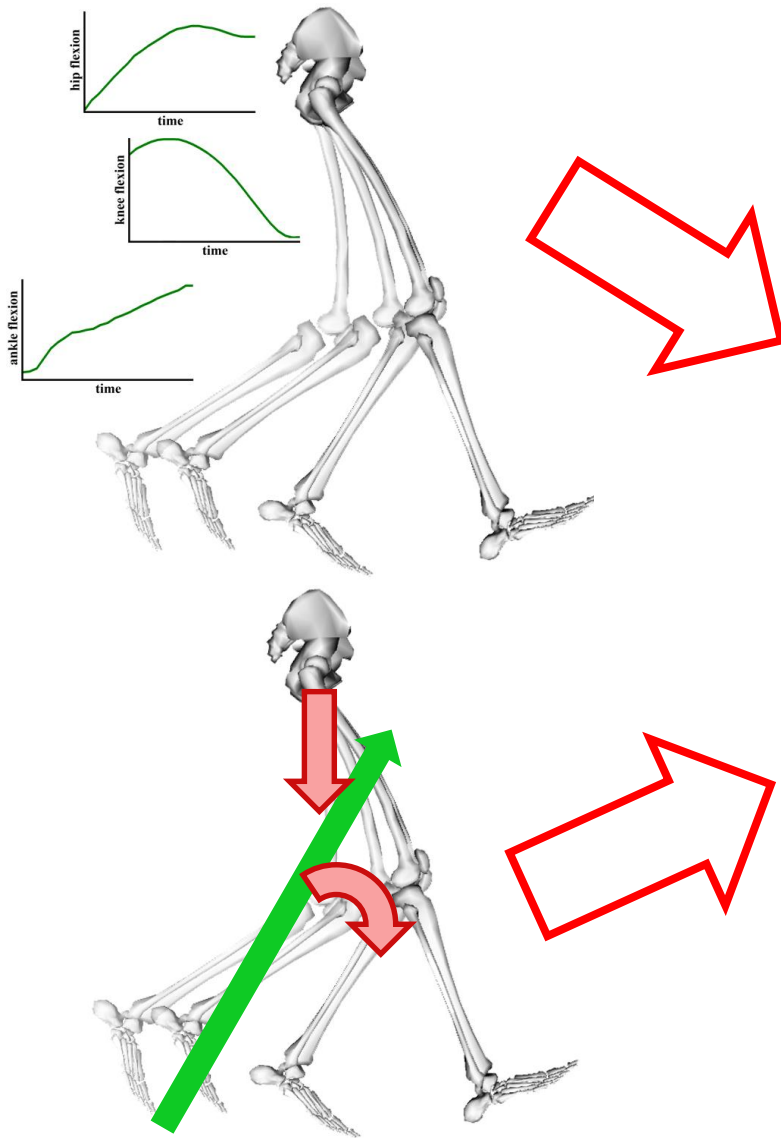
- Kinetics
  - Forces and torques cause the model to accelerate
  - Force
    - Applied to points (e.g., ground reactions) or between points (e.g., muscles)
  - Torque
    - Applied to a coordinate (e.g., joint torque)



# Dynamics: Equations of Motion



# ID: Summary

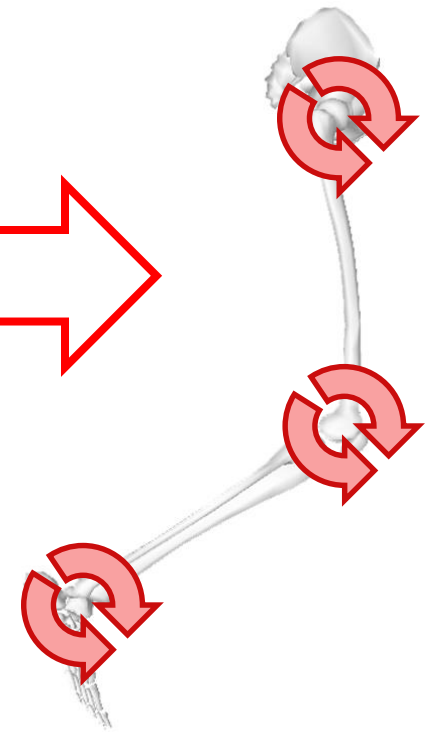


$$\underbrace{\tau}_{\text{unknowns}} = \underbrace{M(q)\ddot{q} - C(q, \dot{q}) - G(q) - F}_{\text{knowns}}$$

**ID**

**Generalized (joint) forces**

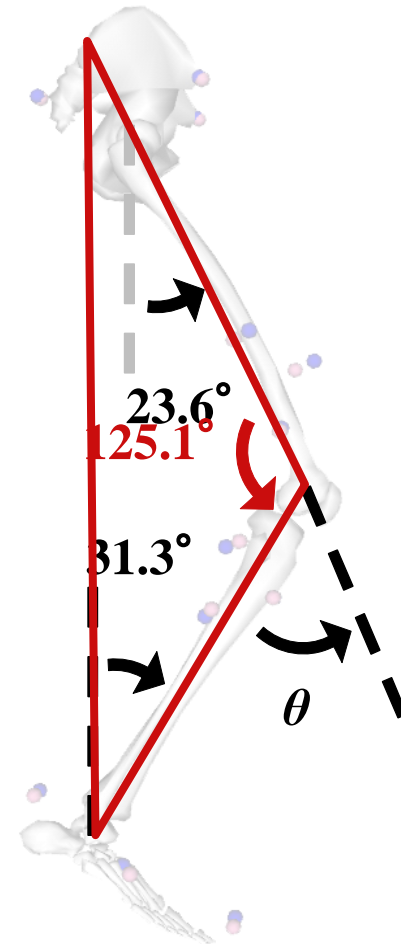
- **NET JOINT FORCES**
- **NET JOINT MOMENTS**



## Exercise

1. For the model shown on the right, what is the **value ( $\theta$ )** of the **knee** coordinate (Note: **extension is +**)?

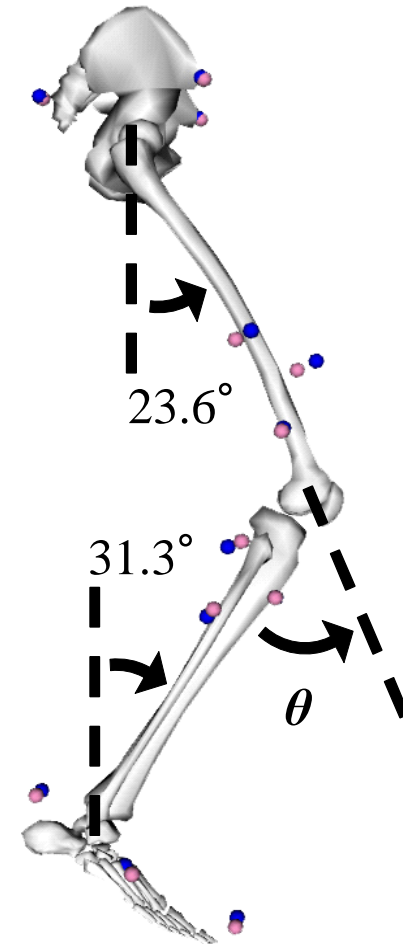
- A.  $23.6^\circ$
- B.  $-54.9^\circ$
- C.  $31.3^\circ$
- D.  $-125.1^\circ$



## Exercise

2. Given that the **model** shown on the right is **at rest**, what is the **velocity** of the knee?

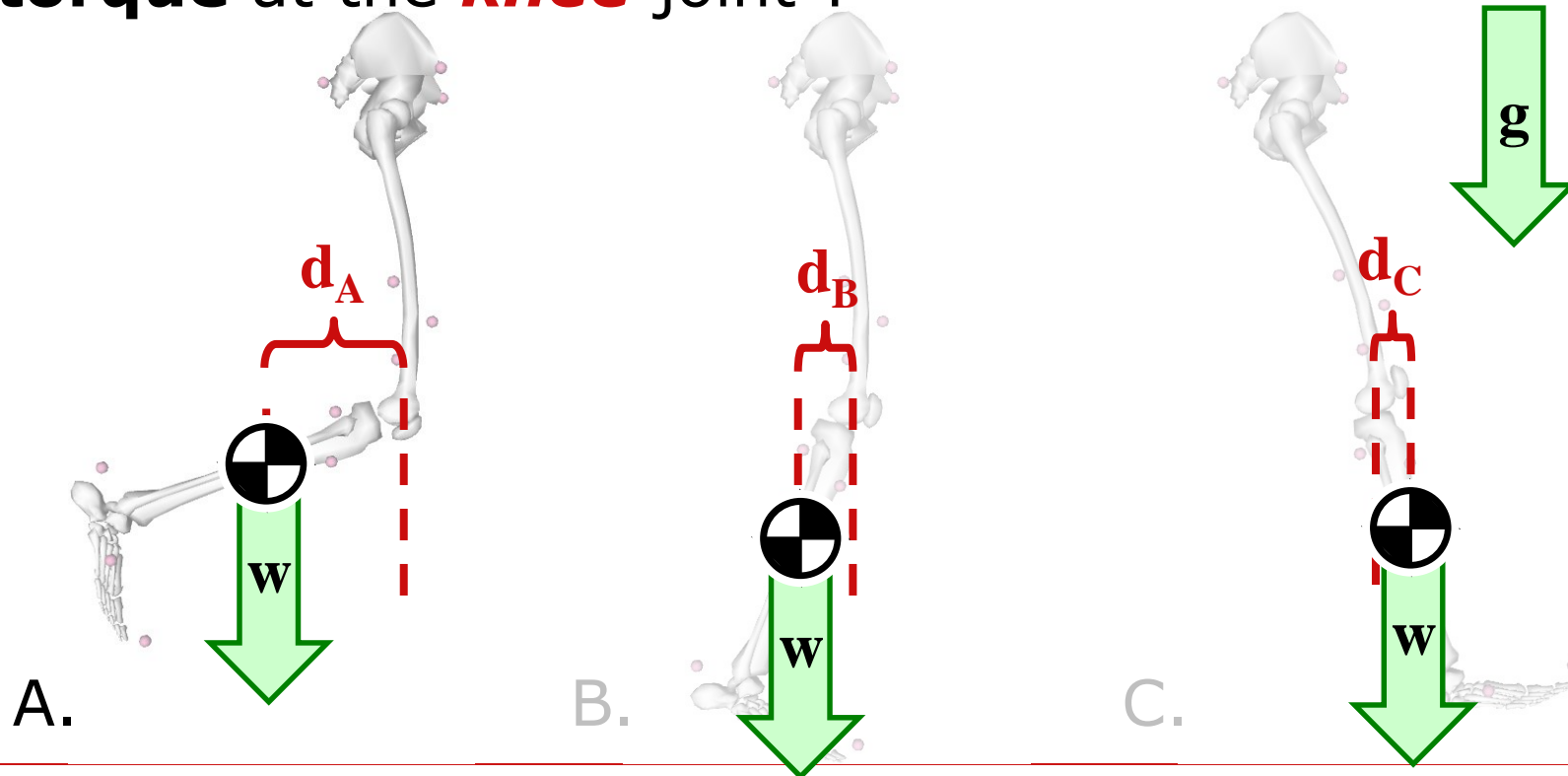
- A.  $23.6^\circ /s$
- B.  $-54.9^\circ /s$
- C.  $3.89^\circ /s$
- D.  $0^\circ /s$



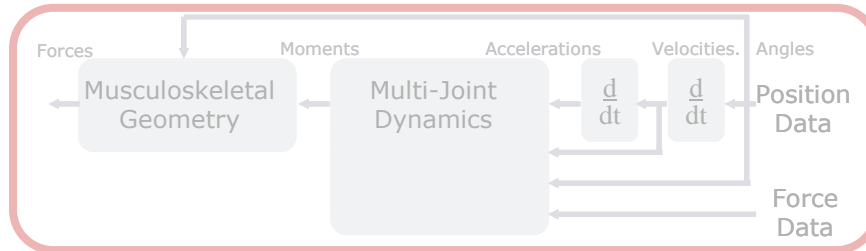


## Exercise

3. For the **model poses** shown below *at rest* and with **gravity ( $g$ )** as the *only force* acting on the model, **which pose** requires the *largest torque* at the *knee* joint ?



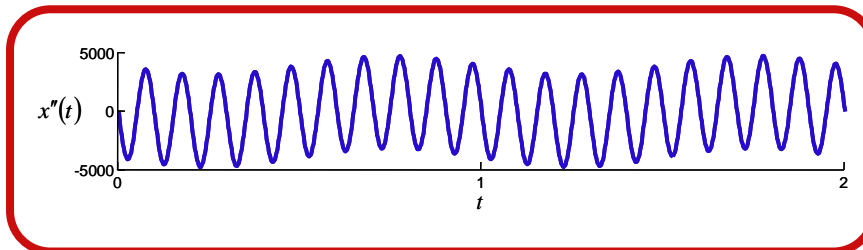
# The Inverse Problem



The inverse problem

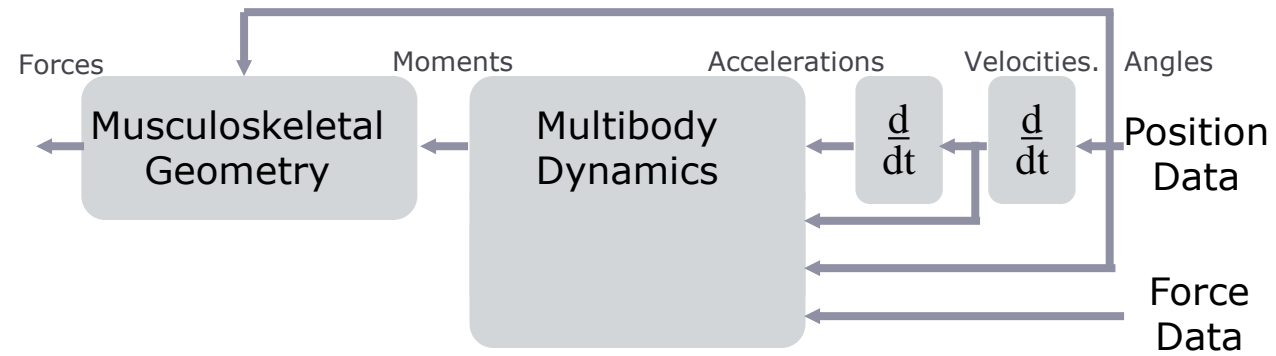


Going from subject motion to joint kinematics  
[Inverse Kinematics]

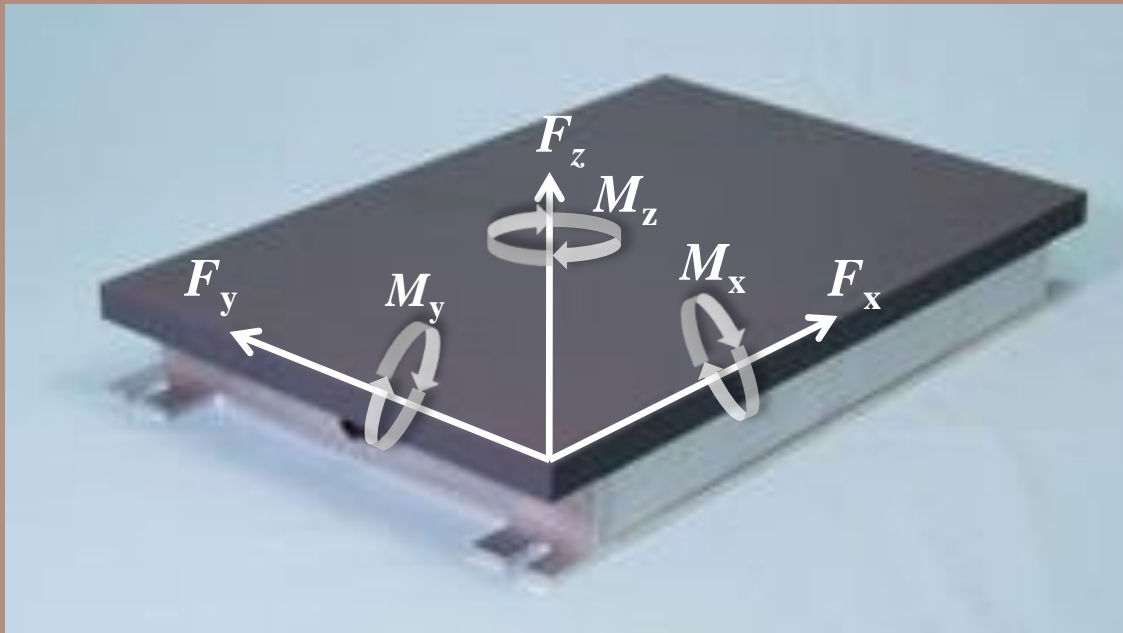
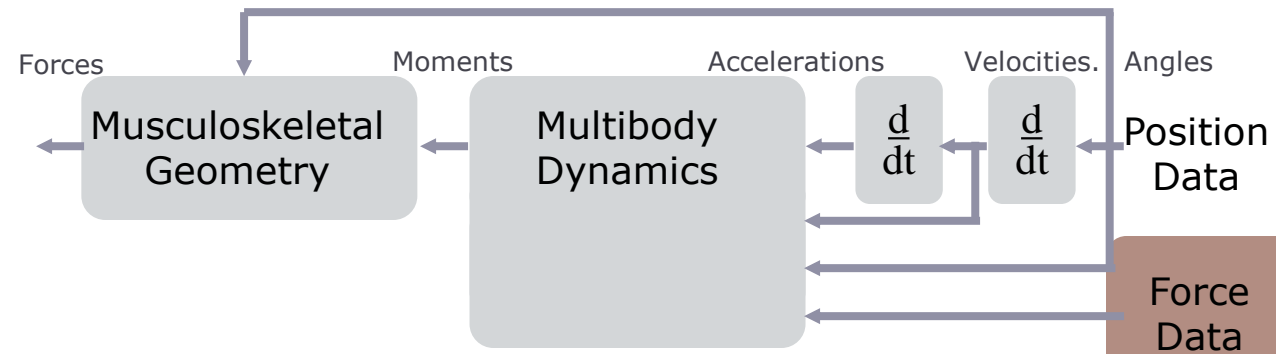


Dealing with noisy data

# The Inverse Problem

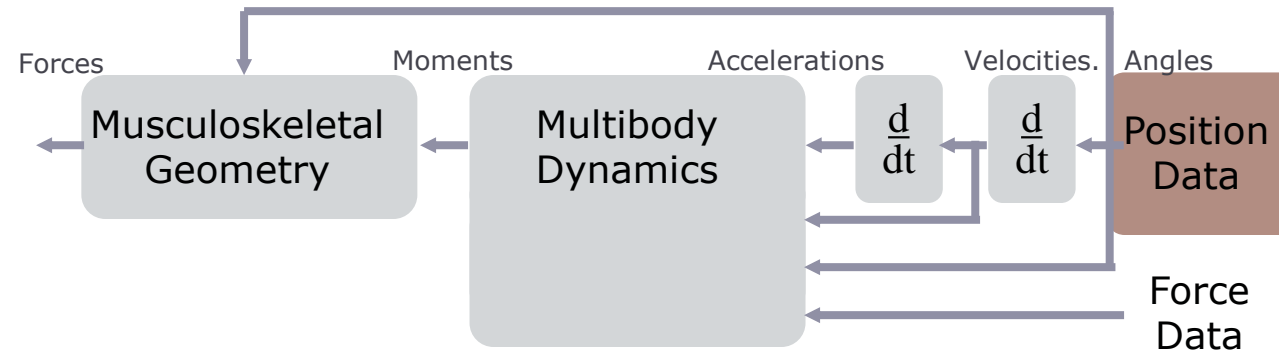


# The Inverse Problem



3 Forces  
3 Moments

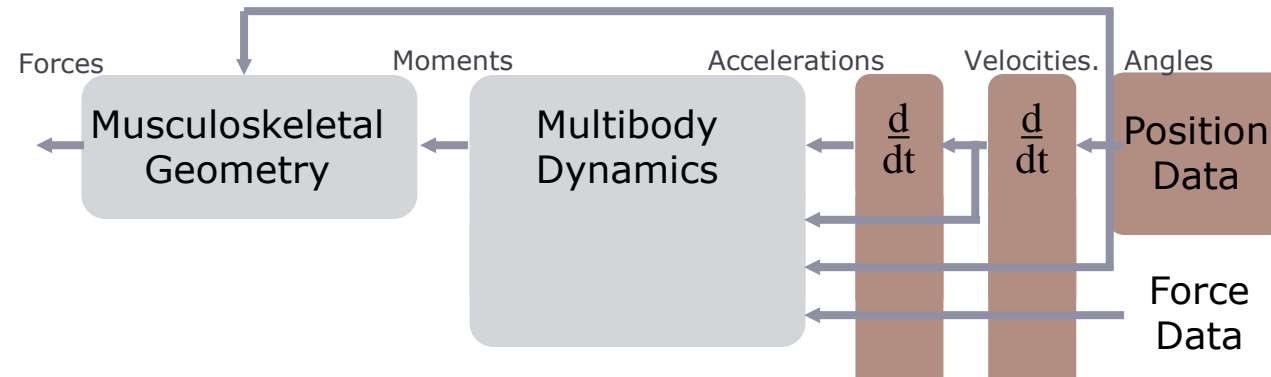
# The Inverse Problem



Video Cameras  
Reflective Markers



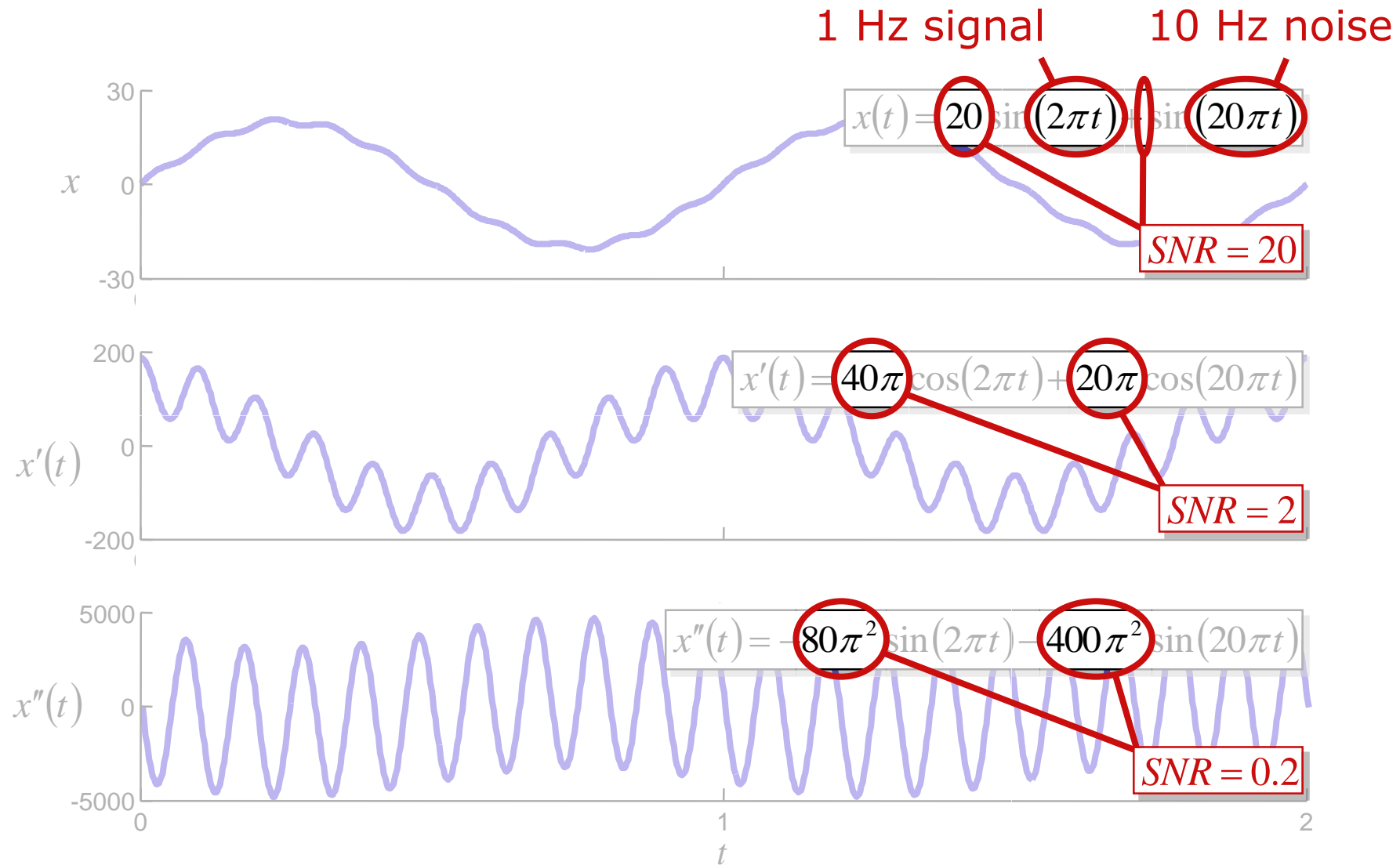
# The Inverse Problem



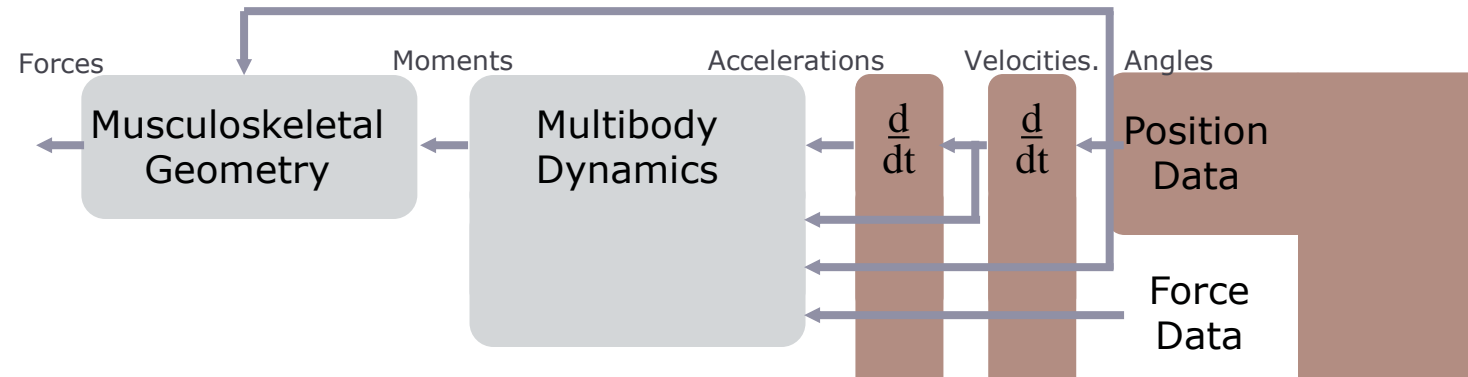
## **Inverse Kinematics**

- Identify research question for the inverse problem
- Determine what should be measured and modeled
- Compute joint kinematics
- Filter and differentiate joint kinematics data

# Differentiation Amplifies High-Frequency Noise



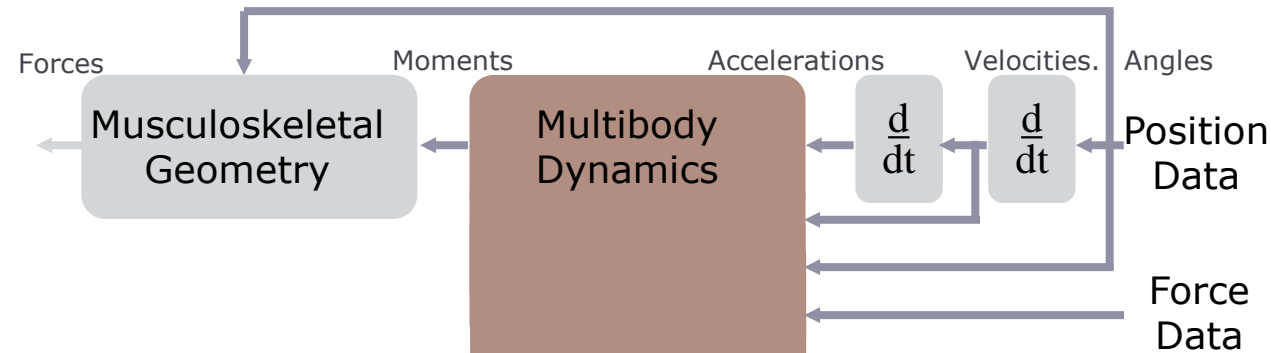
# The Inverse Problem



- ✓ Identified research question for the inverse problem
- ✓ Determined what should be measured and modeled
- ✓ Computed joint kinematics
- ✓ Filtered and differentiated joint kinematics data

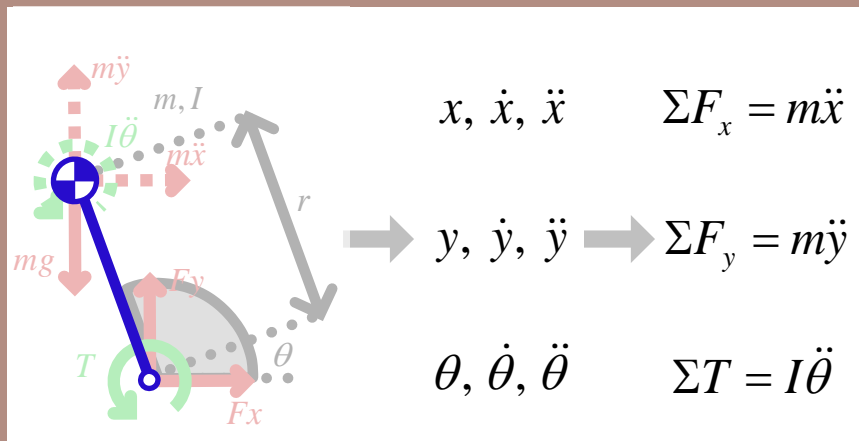


# The Inverse Problem



## Inverse Dynamics

## Inverse Kinematics



- Derive equations of motion defining the model
- Solve equations of motion for joint moments

## A Possible Inverse Dynamics Question

What are the sagittal plane moments about the ankle, knee, and hip during a maximum height jump?

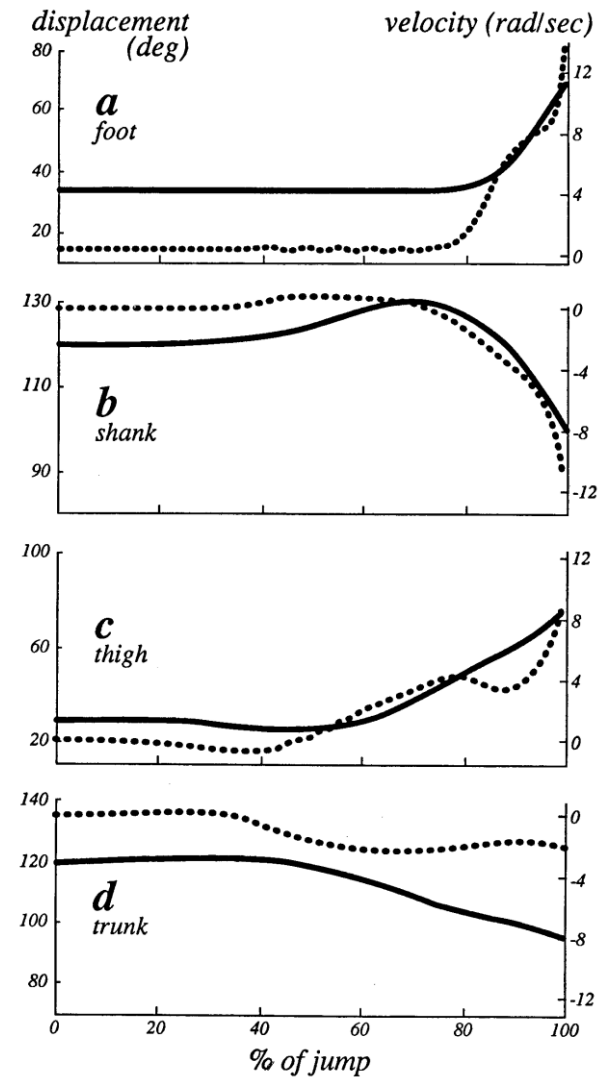
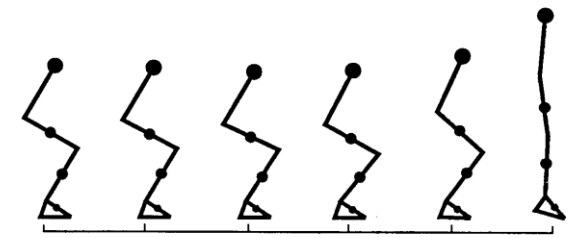
Experimental set-up



# Inverse Dynamics Input: The Experimental Results

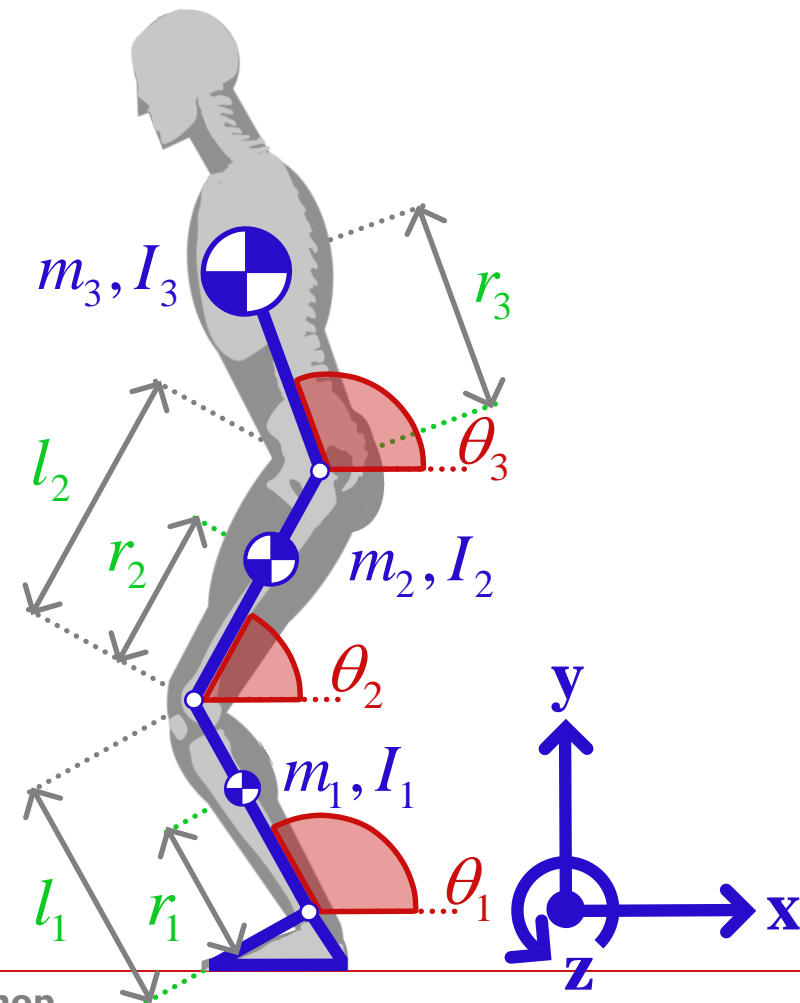
Experiments provide

- joint angles
- angular velocities
- ground reaction forces



# Inverse Dynamics Equations: Multibody Dynamics

- Planar 3 degrees of freedom
- Position (orientation) in global coordinate system
- Segment length =  $l_i$
- Distance to mass center =  $r_i$
- Moments of inertia about mass center
- Foot has no mass and remains on ground



# Inverse Dynamics Equations: Multibody Dynamics

Solved algebraically  
from the ground up

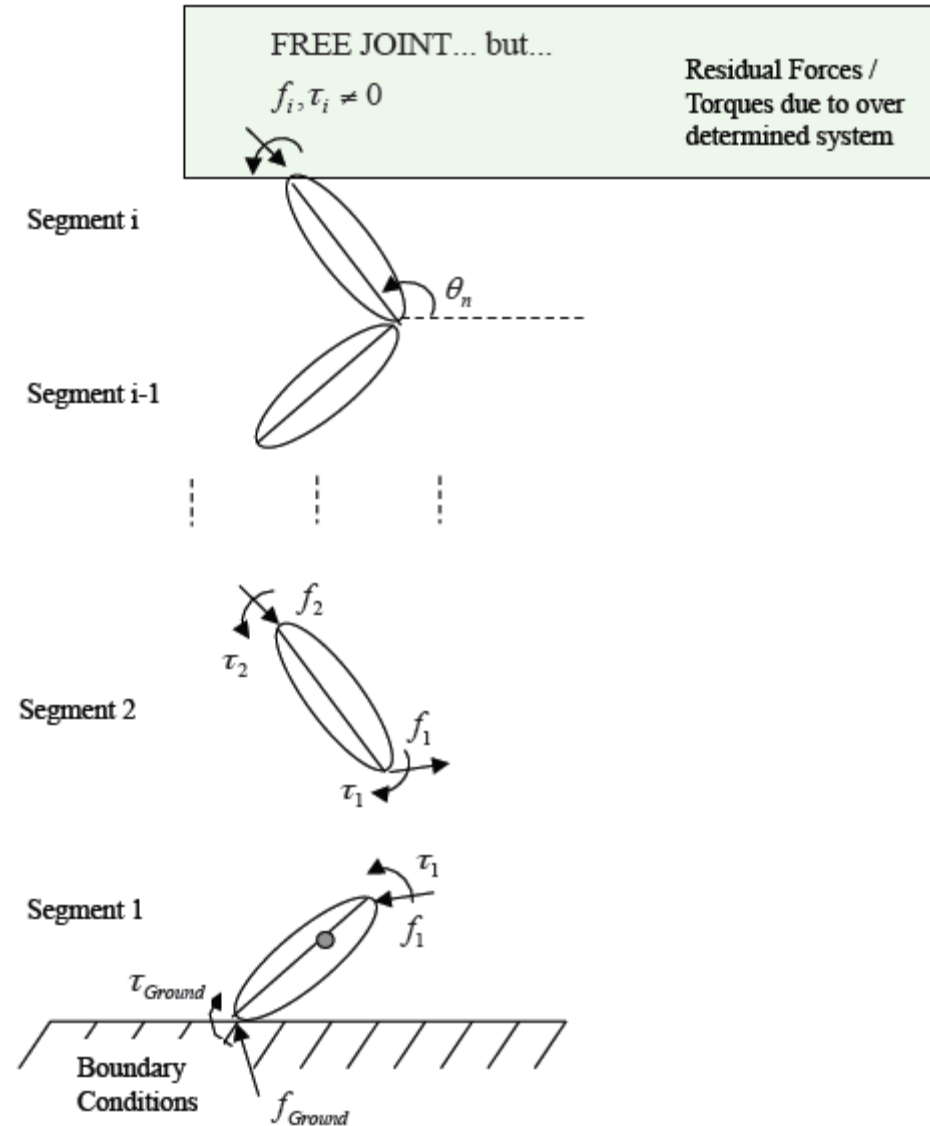
$$x, \dot{x}, \ddot{x} \quad \Sigma F_x = m\ddot{x}$$

$$y, \dot{y}, \ddot{y} \rightarrow \Sigma F_y = m\ddot{y}$$

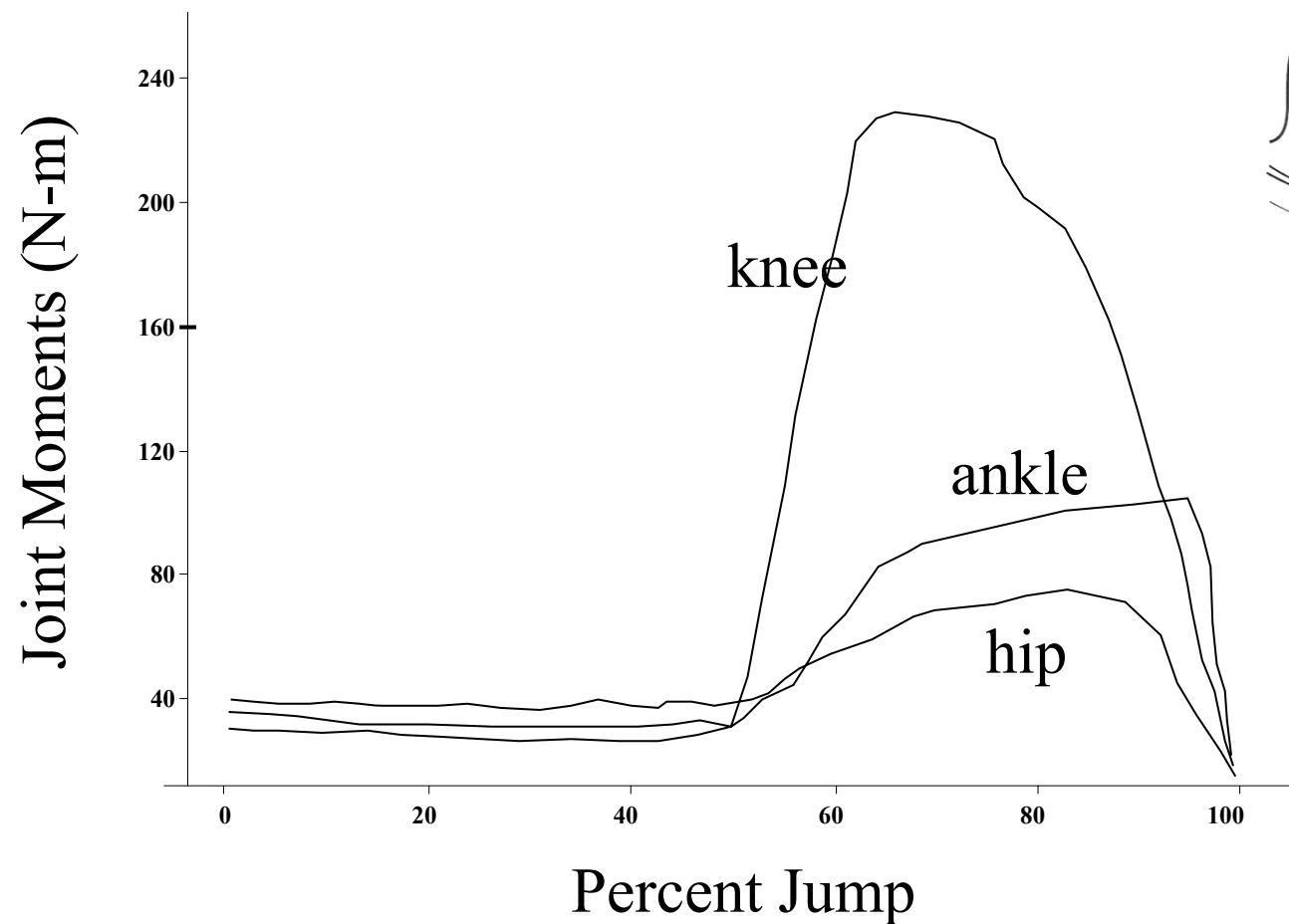
$$\theta, \dot{\theta}, \ddot{\theta} \quad \Sigma T = I\ddot{\theta}$$



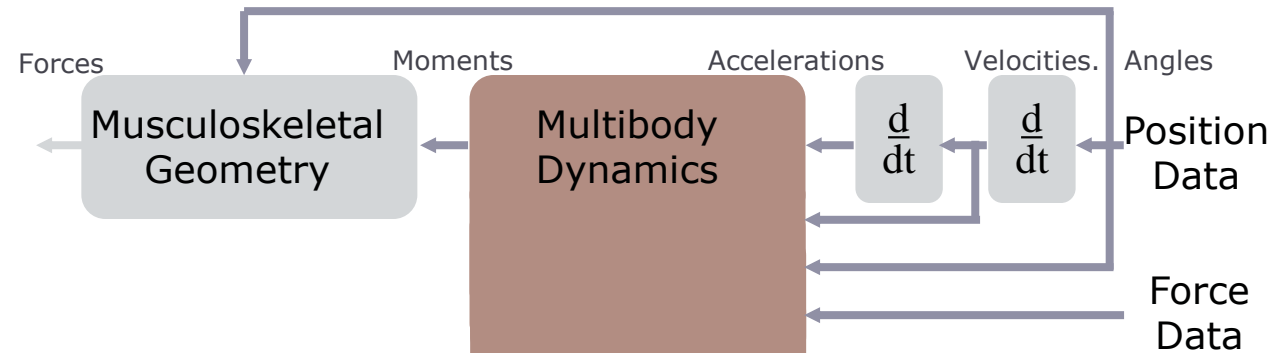
Joint Moments that  
generate the motion



# Inverse Dynamics Output: Net Joint Moments



# The Inverse Problem

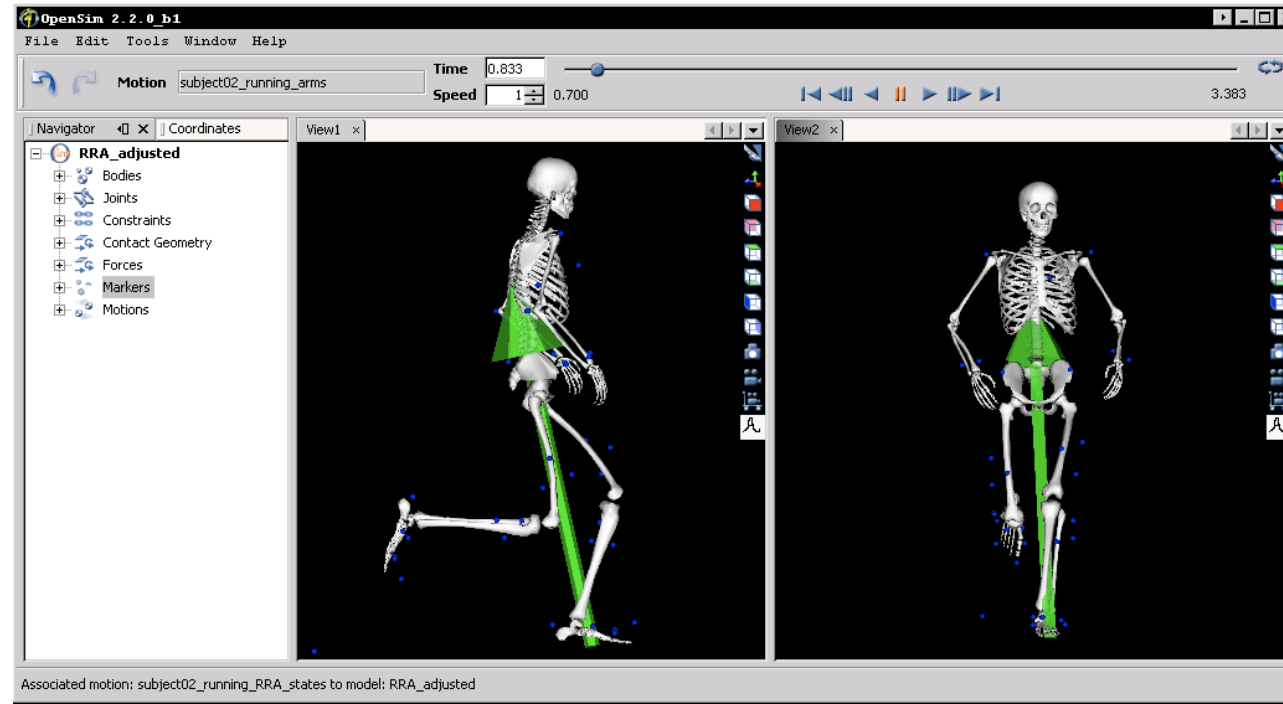


**Inverse  
Dynamics**

**Inverse  
Kinematics**

- ✓ Derived equations of motion defining the model
- ✓ Solved equations of motion for joint moments

# Inverse Dynamics



## **TIPS & TRICKS**

*Filter your raw coordinate data*

*Make sure GRFs were applied correctly and check residuals on the body connected to ground*

*Compare to previous literature data (if available)*