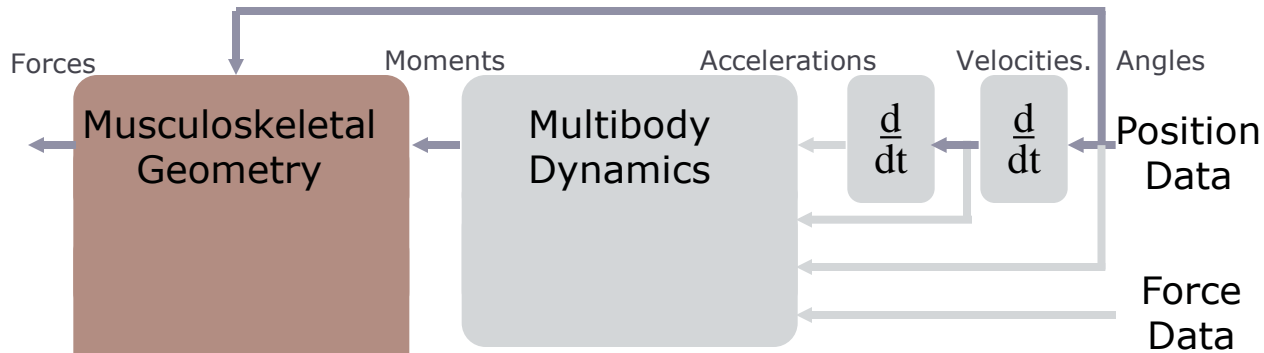


Static Optimization

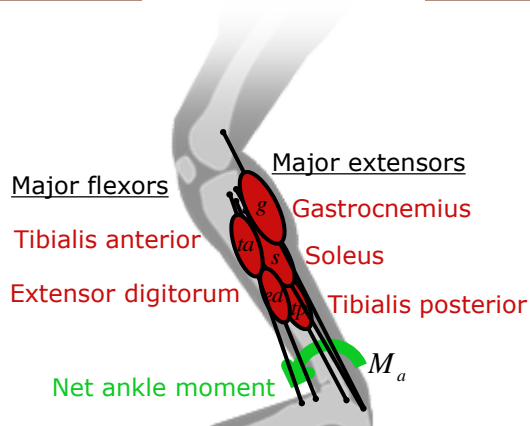
The Inverse Problem



Static Optimization

Inverse Dynamics

Inverse Kinematics



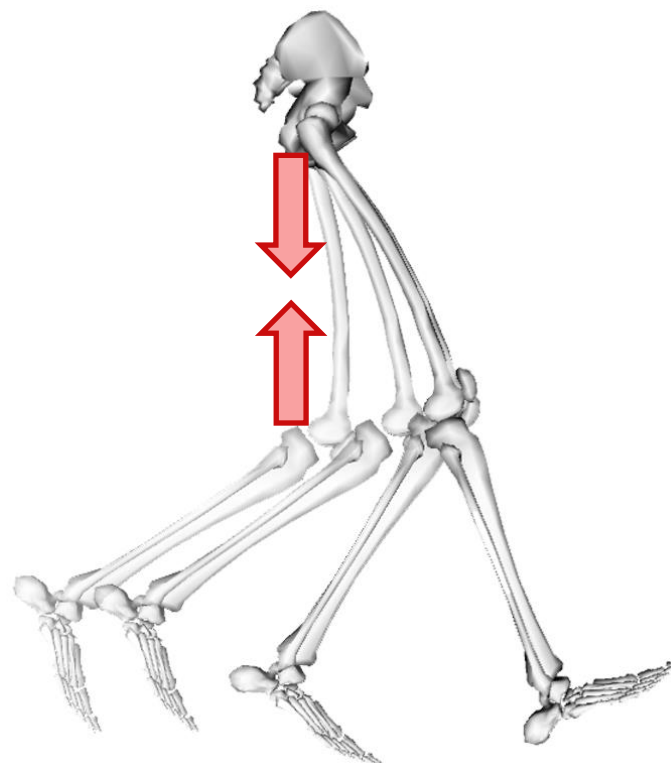
- Use musculoskeletal geometry and assumptions about force distribution to estimate individual muscle forces

Key Concepts

- **Kinematics** coordinates and their velocities and accelerations
- **Kinetics** muscle forces
- **Muscle physiology** muscle activation-contraction dynamics and force-length-velocity relations
- **Dynamics** equations of motion
- **Musculoskeletal geometry** muscle moment arm
- **Optimization** the “distribution” problem

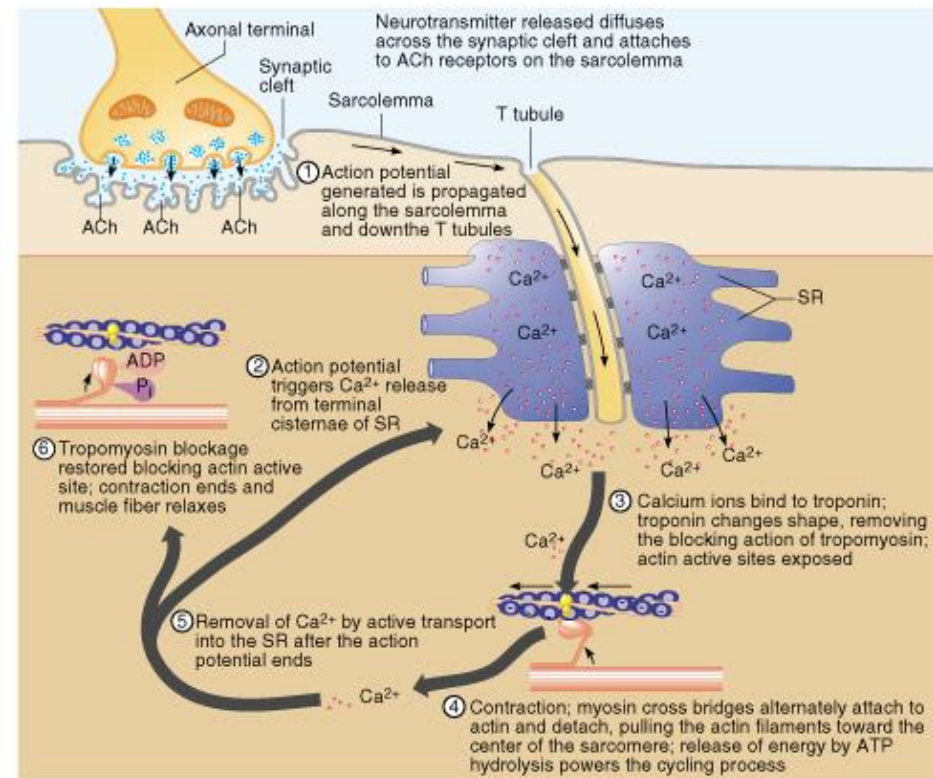
Kinetics: Muscle Forces

- Kinetics
 - Muscle forces cause the model to accelerate
- Muscle force
 - Applied between origin and insertion points



Muscle Physiology: Muscle Activation- Contraction and Force-Length-Velocity Relations

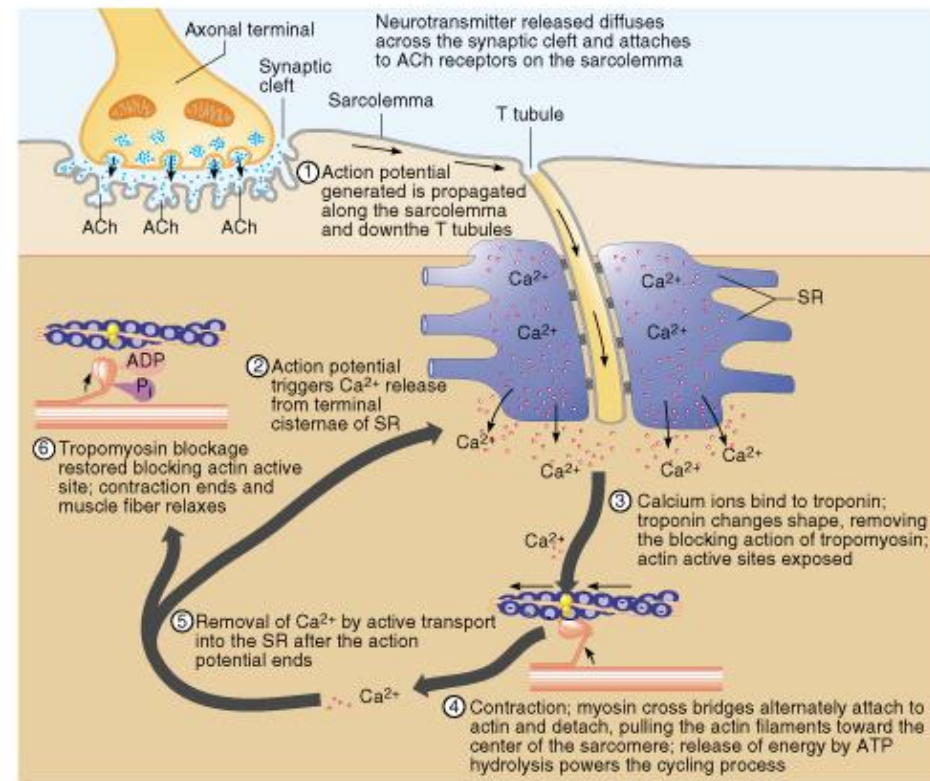
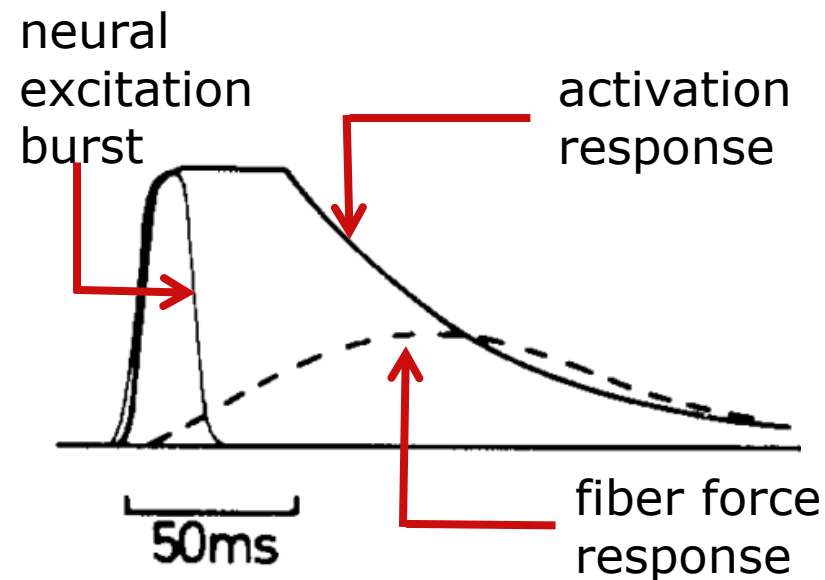
- Muscle activation-contraction
 - Biochemical reaction that causes a muscle's fibers to contract which produces force



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Muscle Physiology: Muscle Activation- Contraction and Force-Length-Velocity Relations

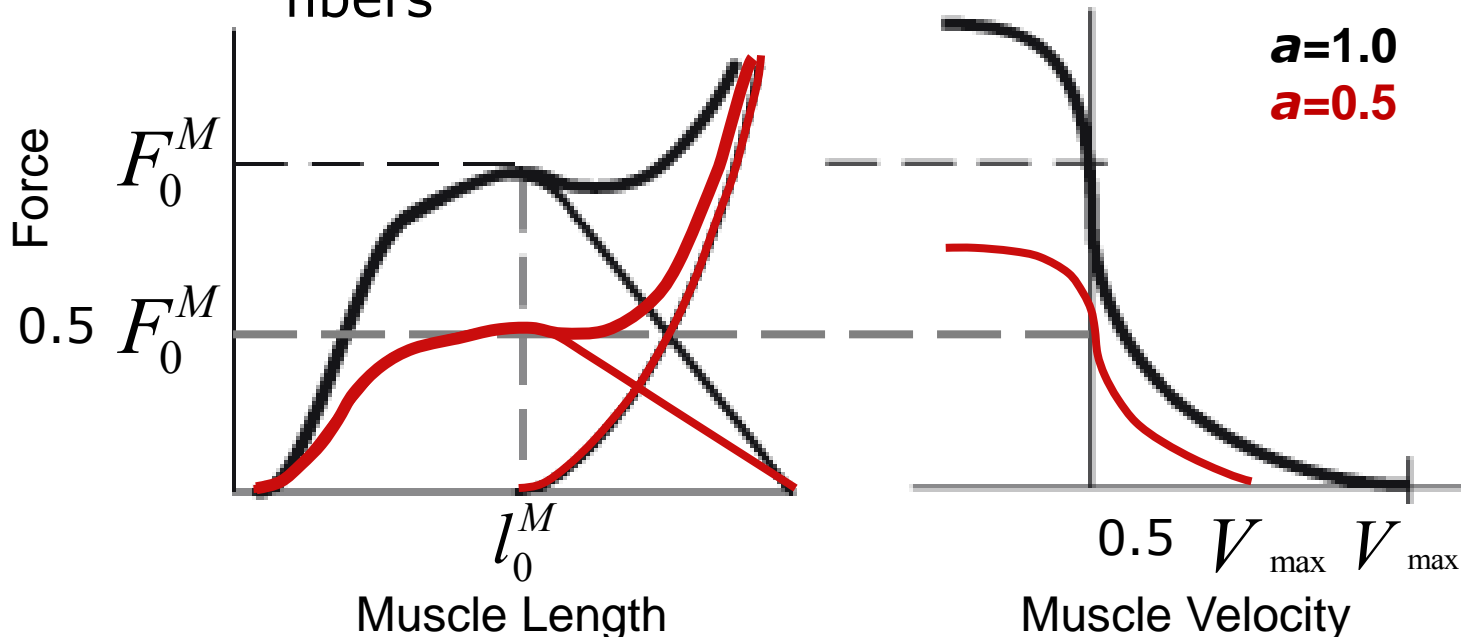
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Muscle Physiology: Muscle Activation-Contraction and Force-Length-Velocity Relations

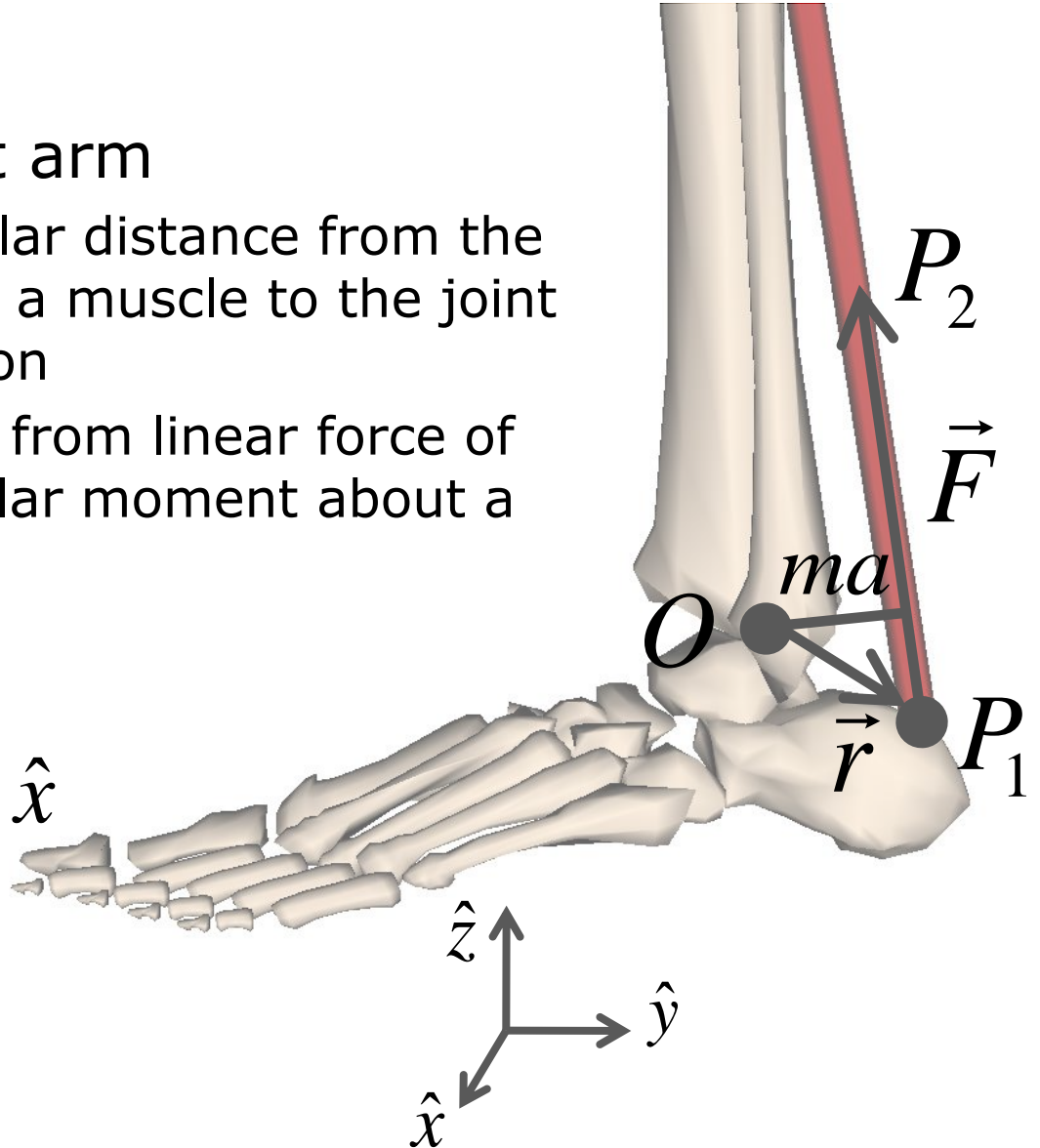
- Muscle activation-contraction
 - Biochemical reaction that causes a muscle's fibers to contract which produces force
- Muscle force-length-velocity
 - Force production diminishes for short, long, and fast fibers



Musculoskeletal Geometry: Muscle Moment Arm

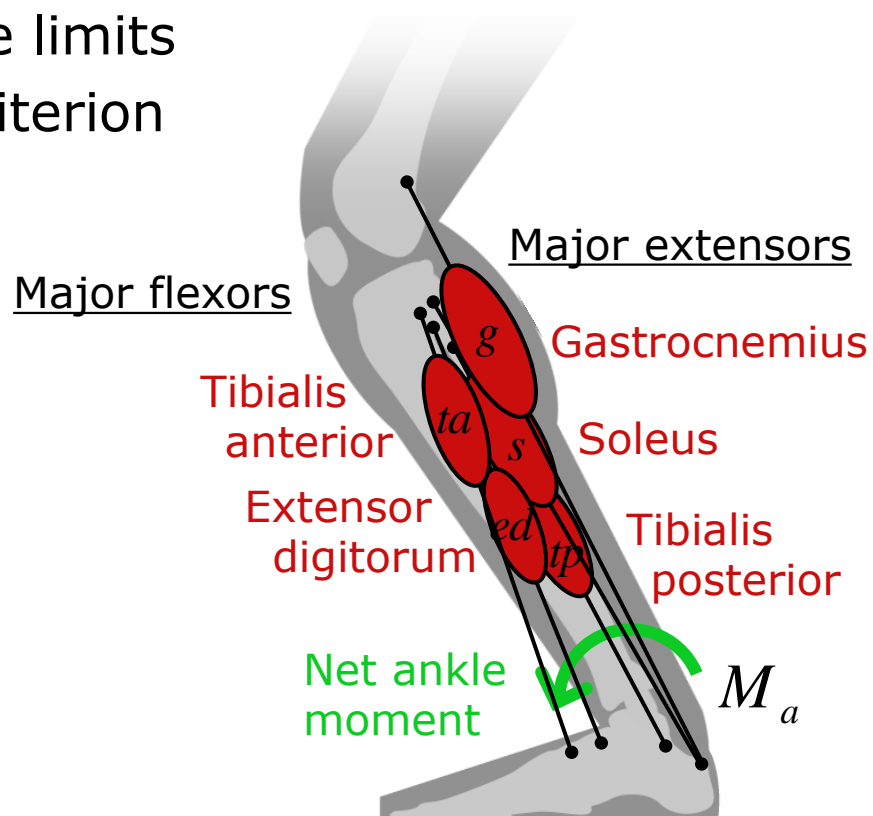
- Muscle moment arm
 - The perpendicular distance from the line of action of a muscle to the joint center of rotation
 - Transformation from linear force of muscle to angular moment about a joint center

$$ma_x = \frac{\vec{r} \times \vec{F}}{|\vec{F}|} \cdot \hat{x}$$



Static Optimization

- Determines the “best” set of muscle forces that
 - Produce net joint moments at a discrete time
 - Do not violate muscle force limits
 - Optimize a performance criterion
- Performance criterion attempts to capture the goal of the neural control system
 - Minimize muscle force?
 - Minimize muscle stress?



The Muscle Force Distribution Problem

$$M_j = \sum \text{muscle moments} + \sum \text{moments due to other structures}$$

number of flexors

number of extensors

$$M_j = \sum_{f=1}^{n_f} F_f r_f - \sum_{e=1}^{n_e} F_e r_e$$

flexion moment

extension moment

moment arm

1 equation with $n_f + n_e$ unknowns

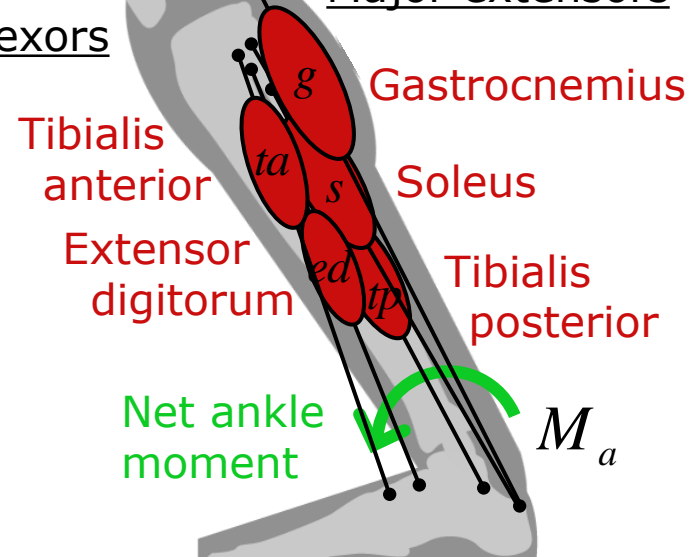
Ankle example

$$M_a = (F_{ta} r_{ta} + F_{ed} r_{ed}) - (F_g r_g + F_s r_s + F_{tp} r_{tp})$$

How can we solve this?

Major flexors

Major extensors



Static Optimization Formulation

minimize $f(F_m)$ Function of muscle forces

subject to $M_a(t) = [F_{ta}(t)r_{ta}(t) + F_{ed}(t)r_{ed}(t)] - [F_g(t)r_g(t) + F_s(t)r_s(t) + F_{tp}(t)r_{tp}(t)]$

$$F_{ta}(t) \leq 900\text{N}$$

$$F_{ed}(t) \leq 800\text{N}$$

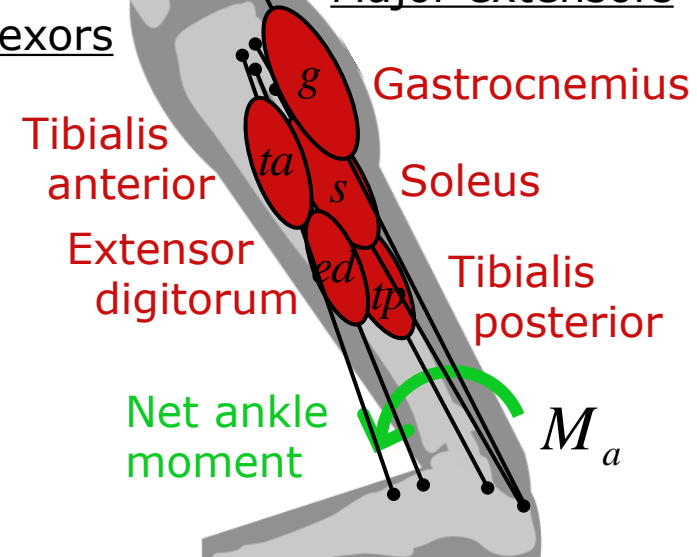
$$F_g(t) \leq 1500\text{N}$$

$$F_s(t) \leq 2500\text{N}$$

$$F_{tp}(t) \leq 1500\text{N}$$

Major flexors

Major extensors



Example Performance Criteria

$$f(F_m) = \sum_{m=1}^{nm} F_m$$

$$f(F_m) = \sum_{m=1}^{nm} \left(\frac{F_m}{PCSA_m} \right)^3$$

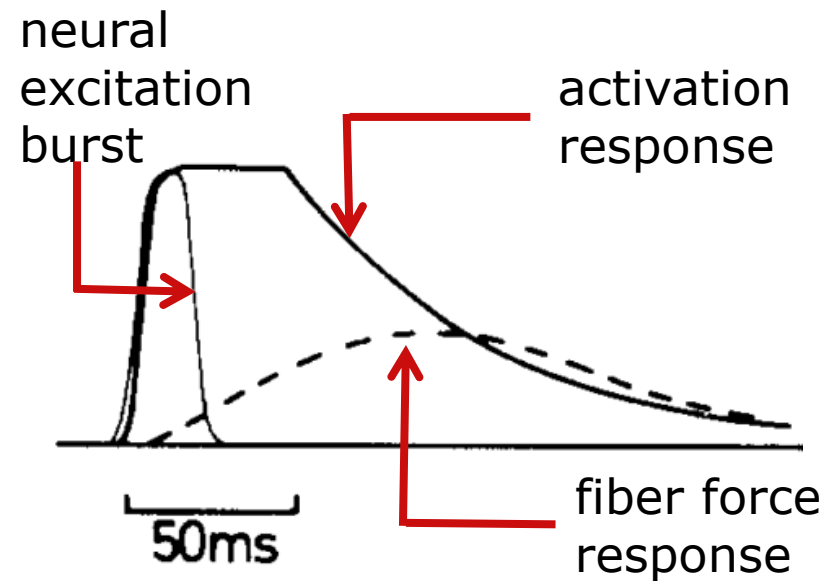
$$f(F_m) = \sum_{m=1}^{nm} \left(k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2$$

Muscle force

Difficult to define and validate a good criterion

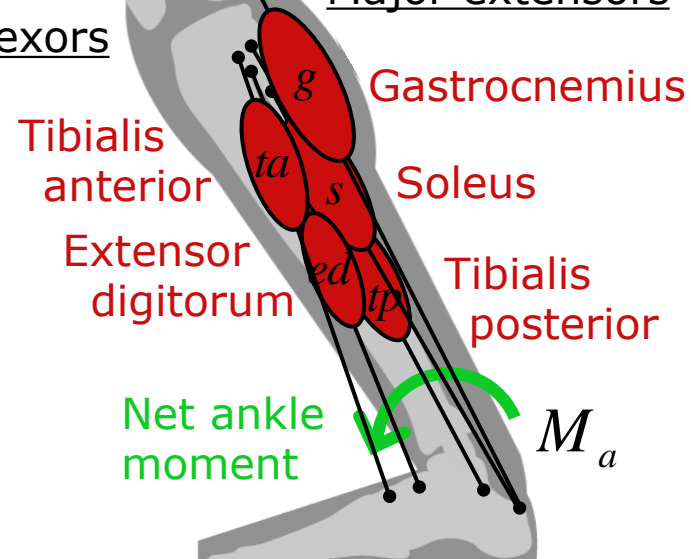
(Muscle stress)³ ~ Metabolic energy

(Muscle activation)²



Major flexors

Major extensors



Example Performance Criteria

$$f(F_m) = \sum_{m=1}^{nm} F_m$$

Muscle force

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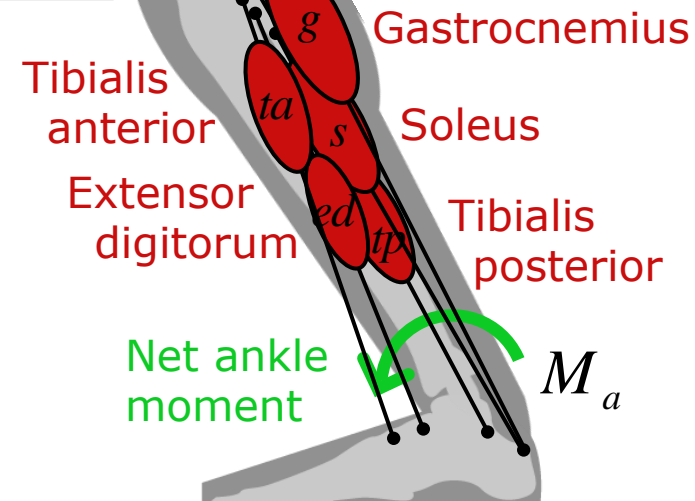
(Muscle activation)²

Possible validations

- Use output to drive a forward dynamic simulation
- Compare qualitatively to experimental EMG
- Compare to measured forces (instrumented hip implant, buckle transducer in tendon)

Major flexors

Major extensors



Example Performance Criteria

$$f(F_m) = \sum_{m=1}^{nm} F_m$$

Muscle force

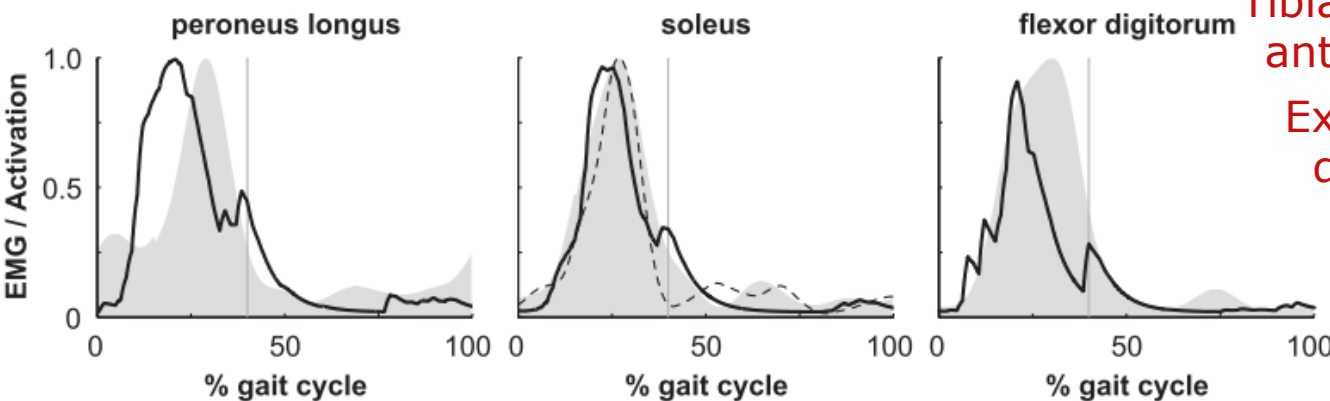
Difficult to define and validate a good criterion

$$f(F_m) = \sum_{m=1}^{nm} \left(\frac{F_m}{PCSA_m} \right)^3$$

(Muscle stress)³ ~ Metabolic energy

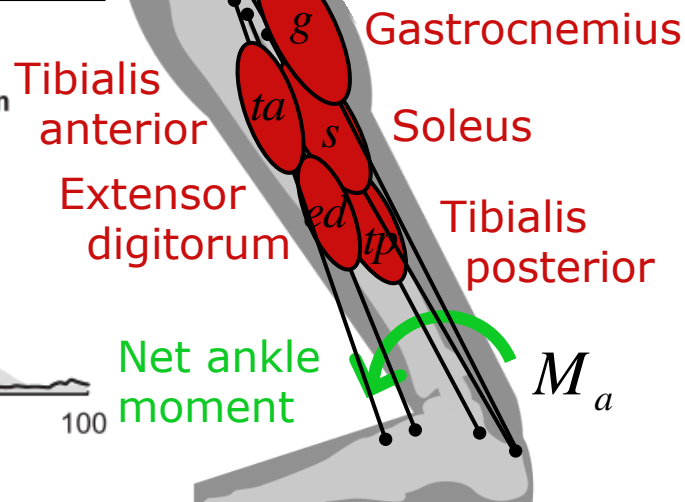
$$f(F_m) = \sum_{m=1}^{nm} \left(k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2$$

(Muscle activation)²



Major flexors

Major extensors



Static Optimization

TIPS & TRICKS

Inputs: Can use kinematics from IK or RRA.
If using IK, need to filter kinematics

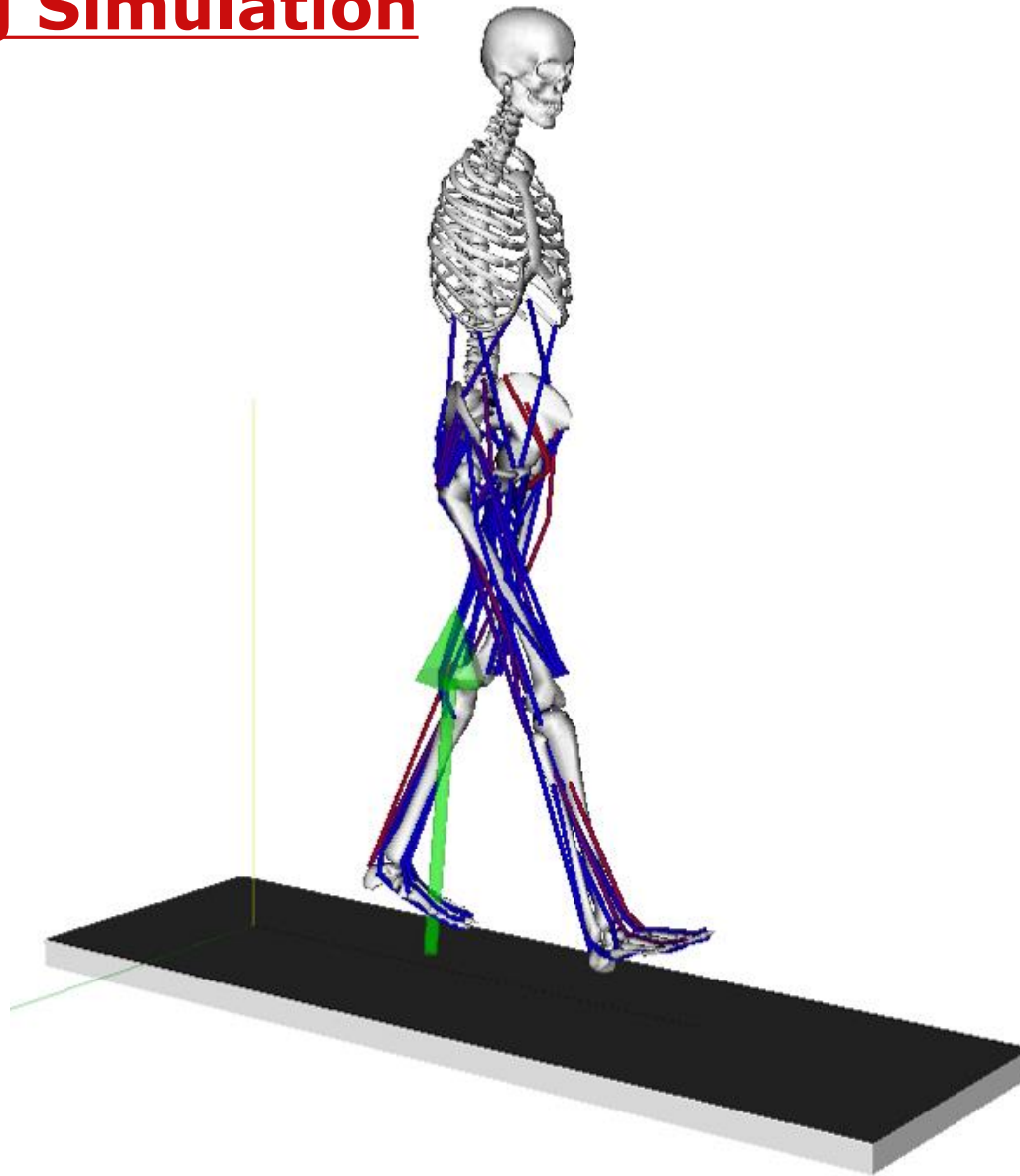
Residuals: Add residual actuators to pelvis

Reserves: Add reserve torque actuators to trouble-shoot a weak model

Minimizing residuals & reserves: Increase maximum control value (default = 1) and lower the maximum force → penalizes activity

Command Line: analyze -S setup_file.xml

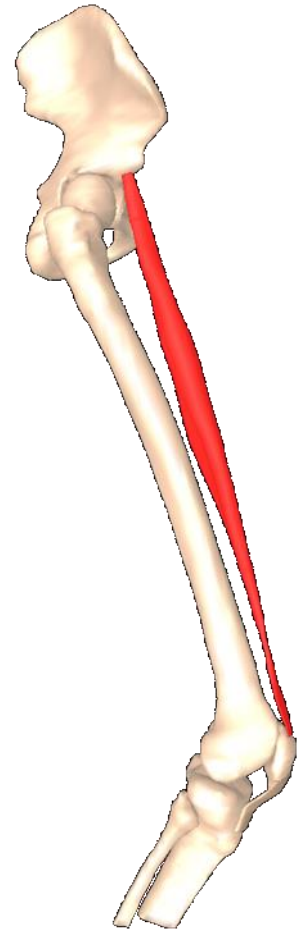
Tracking Simulation



Exercise

1. Given that the rectus femoris muscle has a peak isometric force of 1169 N and it is at its optimal fiber length and zero velocity, what is the force generated for an activation of 0.86?

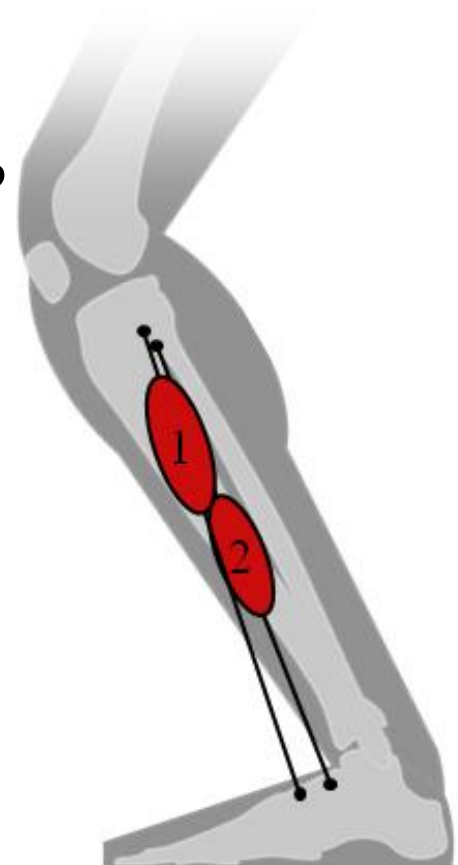
- A. 164 N
- B. 952 N
- C. 1005 N
- D. 1058 N



Exercise

2. For the model shown on the right, which muscle has the largest moment arm about the **ankle** joint?

- A. 1
- B. 2
- C. Neither (are identical)



Exercise

3. For the model shown on the right, which muscle has the largest moment arm about the **knee** joint?
- A. 1
 - B. 2
 - C. Neither (are identical)

