Static Optimization
The Inverse Problem

- Use musculoskeletal geometry and assumptions about force distribution to estimate individual muscle forces.

- Major flexors: aM, Tibialis posterior, Soleus, Gastrocnemius, Major extensors: Extensor digitorum, Tibialis anterior, Net ankle moment $M_a$. 

Static Optimization

Inverse Dynamics

Inverse Kinematics
Key Concepts

• Kinematics coordinates and their velocities and accelerations
• Kinetics muscle forces
• Muscle physiology muscle activation-contraction dynamics and force-length-velocity relations
• Dynamics equations of motion
• Musculoskeletal geometry muscle moment arm
• Optimization the “distribution” problem
**Kinetics: Muscle Forces**

- **Kinetics**
  - Muscle forces cause the model to accelerate
    - Muscle force
      - Applied between origin and insertion points
Muscle Physiology: Muscle Activation-Contraction and Force-Length-Velocity Relations

- Muscle activation-contraction
  - Biochemical reaction that causes a muscle’s fibers to contract which produces force
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Hof, Human Movement Science, 1984
Muscle Physiology: Muscle Activation-Contraction and Force-Length-Velocity Relations

- **Muscle activation-contraction**
  - Biochemical reaction that causes a muscle’s fibers to contract which produces force

- **Muscle force-length-velocity**
  - Force production diminishes for short, long, and fast fibers

\[ F_0^M \]
\[ l_0^M \]
\[ a=1.0 \]
\[ a=0.5 \]

Zajac, Crit Rev Bioeng, 1989
**Musculoskeletal Geometry: Muscle Moment Arm**

- **Muscle moment arm**
  - The perpendicular distance from the line of action of a muscle to the joint center of rotation
  - Transformation from linear force of muscle to angular moment about a joint center

\[
m a_x = \frac{\vec{r} \times \vec{F}}{||\vec{F}||} \cdot \hat{x}
\]
The Muscle Force Distribution Problem

\[ M_j = \sum_{f=1}^{n_f} F_f r_f - \sum_{e=1}^{n_e} F_e r_e \]

where:
- \( M_j \) is the moment due to muscle forces
- \( n_f \) is the number of flexors
- \( n_e \) is the number of extensors
- \( F_f \) is the force of flexor muscles
- \( F_e \) is the force of extensor muscles
- \( r_f \) is the moment arm of flexor muscles
- \( r_e \) is the moment arm of extensor muscles

1 equation with \( n_f + n_e \) unknowns

Ankle example

\[ M_a = (F_{ta} r_{ta} + F_{ed} r_{ed}) - (F_g r_g + F_s r_s + F_{tp} r_{tp}) \]

How can we solve this?
Static Optimization

- Determines the “best” set of muscle forces that
  - Produce net joint moments at a discrete time
  - Do not violate muscle force limits
  - Optimize a performance criterion

- Performance criterion attempts to capture the goal of the neural control system
  - Minimize muscle force?
  - Minimize muscle stress?

- Major flexors:
  - Tibialis posterior
  - Soleus
  - Gastrocnemius

- Major extensors:
  - Extensor digitorum
  - Tibialis anterior

- Net ankle moment: $M_a$
Static Optimization Formulation

minimize $f(F_m)$ 

subject to 

$M_a(t) = [F_{ta}(t)r_{ta}(t) + F_{ed}(t)r_{ed}(t)] - [F_g(t)r_g(t) + F_s(t)r_s(t) + F_{tip}(t)r_{ip}(t)]$

$F_{ta}(t) \leq 900N$

$F_{ed}(t) \leq 800N$

$F_g(t) \leq 1500N$

$F_s(t) \leq 2500N$

$F_{ip}(t) \leq 1500N$
Example Performance Criteria

\[
f(F_m) = \sum_{m=1}^{nm} F_m
\]

\[
f(F_m) = \sum_{m=1}^{nm} \left( \frac{F_m}{PCSA_m} \right)^3
\]

\[
f(F_m) = \sum_{m=1}^{nm} \left( k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2
\]

**Muscle force**

(Muscle stress)\(^3\) ~ Metabolic energy

(Muscle activation)\(^2\)

Difficult to define and validate a good criterion

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**Major flexors**

- Tibialis posterior
- Soleus
- Gastrocnemius

**Major extensors**

- Extensor digitorum
- Tibialis anterior

Net ankle moment

f(\(M_a\))
Example Performance Criteria

Difficult to define and validate a good criterion

\[
f(F_m) = \sum_{m=1}^{nm} F_m \quad \text{Muscle force}
\]

\[
f(F_m) = \sum_{m=1}^{nm} \left( \frac{F_m}{PCSA_m} \right)^3 \quad \text{(Muscle stress)}^3 \sim \text{Metabolic energy}
\]

\[
f(F_m) = \sum_{m=1}^{nm} \left( k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2 \quad \text{(Muscle activation)}^2
\]

Possible validations
- Use output to drive a forward dynamic simulation
- Compare qualitatively to experimental EMG
- Compare to measured forces (instrumented implant, buckle transducer in tendon)

Major flexors
- Tibialis posterior
- Soleus
- Gastrocnemius

Major extensors
- Extensor digitorum
- Tibialis anterior

Net ankle moment
\[ M_a \]
Example Performance Criteria

\[ f(F_m) = \sum_{m=1}^{nm} F_m \]  \hspace{0.5cm} \text{Muscle force}

\[ f(F_m) = \sum_{m=1}^{nm} \left( \frac{F_m}{PCSA_m} \right)^3 \]  \hspace{0.5cm} (Muscle stress)^3 \sim \text{Metabolic energy}

\[ f(F_m) = \sum_{m=1}^{nm} \left( k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2 \]  \hspace{0.5cm} (Muscle activation)^2

Difficult to define and validate a good criterion

\[ \sum = \sum \]

Muscle force

Hamner and Delp, J. Biomech., 2010
# Static Optimization

<table>
<thead>
<tr>
<th>TIPS &amp; TRICKS</th>
</tr>
</thead>
</table>
| **Inputs:** Can use kinematics from IK or RRA.  
  If using IK, need to filter kinematics |
| **Residuals:** Add residual actuators to pelvis |
| **Reserves:** Add reserve torque actuators to trouble-shoot a weak model |
| **Minimizing residuals & reserves:** Increase maximum control value (default = 1) and lower the maximum force → penalizes activity |
| **Command Line:** analyze –S setup_file.xml |
Exercise

1. Given that the rectus femoris muscle has a peak isometric force of 1169 N and it is at its optimal fiber length and zero velocity, what is the force generated for an activation of 0.86?

A. 164 N  
B. 952 N  
C. 1005 N  
D. 1058 N
Exercise

2. For the model shown on the right, which muscle has the largest moment arm about the ankle joint?

A. 1
B. 2
C. Neither (are identical)
Exercise

3. For the model shown on the right, which muscle has the largest moment arm about the **knee** joint?

   A. 1
   B. 2
   C. Neither (are identical)
Exercise

4. For the model shown on the right, muscle 1 and 2 have the following properties

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Peak Isometric Force (N)</th>
<th>Moment Arm (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>905</td>
<td>3.6</td>
</tr>
<tr>
<td>2</td>
<td>512</td>
<td>3.0</td>
</tr>
</tbody>
</table>

To solve the “distribution” problem minimizing the sum of squared activations, which muscle would be activated more for a given dorsiflexion moment?

A. 1  
B. 2  
C. Neither (are identical)
Tracking Simulation