Residual Reduction Algorithm (RRA)

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1 Introduction

2 Reducing Residuals

Throughout this paper, when we use the term “reduce a residual”, or “reduce” a DC offset, it really means “try to eliminate” a residual or DC offset. That is, our strategy is to compute parameter alterations that would, in theory, completely eliminate the DC offset of a particular residual. But since the theoretical solutions we compute are not entirely true in practice since there are many factors affecting a particular residual other than just the one parameter we are altering. However, this failure to completely eliminate the DC offset is actually a good thing: we know that there are unmodeled forces in our system (for example, our model has no arms), so we actually do want some small DC offsets to remain for our residuals just to make us feel like we haven’t eliminated these unmodeled forces.

To reduce a particular residual $R$ with DC offset $d_R \in \mathbb{R}$ by altering a particular parameter $p$ by an amount $\Delta p$, we must compute $\Delta p$ using the following equation:

$$R_{old} - R_{new} = d_R$$

We require that $R_{new}$ and $R_{old}$ be expressible in terms of inertial parameters and possibly joint variables, and that $R_{new}$ also be expressed in terms of $\Delta p$. Then the above equation can be solved for $\Delta p$ in terms of the inertial parameters and the DC offset $d_R$. It is easier to see why this equation is true if we look at it this way:

$$R_{new} = R_{old} - d_R$$

Here, $R_{old}$ is the original residual with the DC offset $d_R$. If we remove the DC offset from $R_{old}$, i.e. if we subtract the DC offset from $R_{old}$, we get $R_{new}$.

2.1 Reducing Forward-Backward Rocking

We will reduce the residual $MZ$ by independently altering two parameters: the torso center of mass $x$-coordinate by an amount $\Delta t_x$ and the lumbar extension angle by an amount $\Delta l_e$.

2.1.1 Altering the Torso Center of Mass

Here we will compute an amount $\Delta t_x$ by which to alter the $x$-coordinate of the torso center of mass in order to balance the DC offset of the $MZ$ residual. Let $m$ be the mass of the torso and let $g$ denote acceleration due to gravity. Let $d_{MZ}$ be the DC offset of the $MZ$ residual. Let $r_0$ be the moment arm (lever arm) of the torso, which we define to be the vector pointing from the pelvis center of mass to the torso center of mass. Note that $r_0$ varies as the torso position varies, but its magnitude stays fixed. Then we have that the original value of $MZ$ at any torso position is:

$$MZ_{old} = r_0 \times mg$$
Let \( r_1 \) be the torso moment arm after the center of mass has been displaced in the \( x \) direction by \( \Delta t_x \). Note that \( r_1 \) may not have the same magnitude as \( r_0 \). Then the new value of \( MZ \) is:

\[
MZ_{\text{new}} = r_1 \times mg = (r_0 + (\Delta t_x, 0, 0)) \times mg = r_0 \times mg + (\Delta t_x, 0, 0) \times mg
\]

The last step is correct since the cross product distributes over addition. Let \( d_{MZ} = (0, 0, d_{MZ}) \), i.e. \( d_{MZ} \) is a vector representation of the DC offset. Now we plus the above expressions into the equation \( MZ_{\text{old}} - MZ_{\text{new}} = d_{MZ} \):

\[
r_0 \times mg - (r_0 \times mg + (\Delta t_x, 0, 0) \times mg) = d_{MZ}
\]

The \( r_0 \times mg \) expressions cancel out on both sides, and the value of the remaining cross product is

\[
(\Delta t_x, 0, 0) \times mg = \begin{vmatrix} i & j & k \\ \Delta t_x & 0 & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta t_x).
\]

So we are left with

\[-(0, 0, -mg\Delta t_x) = (0, 0, d_{MZ})\]

or looking at just the \( z \)-coordinates

\[mg\Delta t_x = d_{MZ}\]

\[\Delta t_x = \frac{d_{MZ}}{mg}.
\]

(1)

So, in order to reduce the DC offset of the \( MZ \) residual, i.e. to reduce the average forward-backward rocking motions of a walking model, our computation suggests that we should alter the torso center of mass \( x \)-coordinate by an amount \( \frac{d_{MZ}}{mg} \).

2.1.2 Altering the Lumbar Extension Angle

Now we wish to compute an amount \( \Delta l_e \) by which to alter the lumbar extension angle (throughout the entire time interval, not just at the initial time) so that the DC offset for \( MZ \) is reduced. We can represent the alteration of the lumbar extension angle with the following geometry: consider the triangle consisting of two vectors \( r_0 \) and \( r_1 \) with equal length \( r_0 \) and with a common starting point with an angle \( \Delta l_e \) between them. Suppose the vectors are oriented so that \( \Delta l_e \) is drawn in a positive sense (counterclockwise) when it is drawn from \( r_0 \) to \( r_1 \). Let \( \Delta l = r_1 - r_0 \). Assuming \( \Delta l_e \) is small, we can apply an approximation from biomechanics which states that the moment arm (lever arm) of a muscle is equal to \( \delta l / \delta \theta \) where \( \delta l \) is the change in length of the muscle when the joint spanned by the muscle rotates by a small angle \( \delta \theta \). Applying this approximation to our triangle, we have that

\[
\Delta l = r_0 + \Delta l
\]

where \( \Delta l = \| \Delta l \| \). We will show how to compute the (direction of) the vector \( \Delta l \) later. As before, we define \( MZ_{\text{new}} = r_1 \times mg \) and \( MZ_{\text{old}} = r_0 \times mg \). From the definition of \( \Delta l \), we know that \( r_1 = r_0 + \Delta l \). Plugging into the equation \( MZ_{\text{old}} - MZ_{\text{new}} = d_{MZ} \), we have

\[
r_0 \times mg - (r_0 \times mg + \Delta l \times mg) = (0, 0, d_{MZ})
\]

\[-\Delta l \times mg = (0, 0, d_{MZ}).
\]

If we write \( \Delta l = (\Delta l_x, \Delta l_y, 0) \), then we have

\[
\Delta l \times mg = \begin{vmatrix} i & j & k \\ \Delta l_x & \Delta l_y & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta l_x).
\]

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Subsituting into the previous equation, we have

\[ mg\Delta l = d_{MZ}. \]

Now if we write \( \Delta l = \Delta l \cos \theta \) where \( \theta \) is the angle representing the orientation of the vector \( \Delta l \) relative to the positive \( x \)-axis (counterclockwise is positive), and since we know the length of the vector is \( \Delta l = r_0 \Delta l_e \), we have that \( \Delta l_e = r_0 \Delta l_e \cos \theta \), so substituting into the above equation yields

\[ mg r_0 \Delta l_e \cos \theta = d_{MZ}. \]

Let \( \alpha \) be the angle representing the orientation of \( r_0 \), measured in a positive (counterclockwise) sense starting from the positive \( x \)-axis. The angle \( \Delta l \) is the orientation of \( r_0 \) as measured in a positive (counterclockwise) sense starting from the positive \( y \)-axis. So \( \alpha = \Delta l_e + 90^\circ \). Since we assumed that \( \Delta l_e \) is small, the vector \( \Delta l \) is approximately tangent to the circle with radius \( r_0 \) centered at the pelvis center of mass, i.e. we can assume that \( \Delta l \) is just \( r_0 \) rotated counterclockwise by \( 90^\circ \) and scaled. Since we defined \( \theta \) to be the angle swept counterclockwise from the positive \( x \)-axis to \( \Delta l \), then we can assume that \( \theta = \alpha + 90^\circ = \Delta l_e + 180^\circ \).

Hence we have

\[ \cos \theta = \cos(\Delta l_e + 180^\circ) = \cos \Delta l_e \cos 180^\circ - \sin \Delta l_e \sin 180^\circ = -\cos \Delta l_e \]

so

\[ \Delta l_e = \frac{d_{MZ}}{mgr_0 \cos \Delta l_e}. \]

**2.2 Reducing Left-Right Rocking**

We will reduce the residual \( M_X \) by independently altering two parameters: the torso center of mass \( z \)-coordinate by an amount \( \Delta t_z \) and the lumbar bending angle by an amount \( \Delta l_b \).

**2.2.1 Altering the Torso Center of Mass**

Here we will compute an amount \( \Delta t_z \) by which to alter the \( z \)-coordinate of the torso center of mass in order to balance the DC offset of the \( M_X \) residual. Let \( d_{M_X} \) be the DC offset of the \( M_X \) residual. The original value of \( M_X \) at any torso position is:

\[ M_{X,old} = r_0 \times mg \]

Let \( r_1 \) be the torso moment arm after the center of mass has been displaced in the \( z \) direction by \( \Delta t_z \). Note that \( r_1 \) may not have the same magnitude as \( r_0 \). Then the new value of \( M_X \) is:

\[ M_{X,new} = r_1 \times mg \]

\[ = (r_0 + (0, 0, \Delta t_z)) \times mg \]

\[ = r_0 \times mg + (0, 0, \Delta t_z) \times mg \]

Let \( d_{M_X} = (d_{M_X}, 0, 0) \), i.e. \( d_{M_X} \) is a vector representation of the DC offset. Now we plus the above expressions into the equation \( M_{X,old} - M_{X,new} = d_{M_X} \):

\[ r_0 \times mg - (r_0 \times mg + (0, 0, \Delta t_z) \times mg) = d_{M_X} \]

The \( r_0 \times mg \) expressions cancel out on both sides, and the value of the remaining cross product is

\[ (0, 0, \Delta t_z) \times mg = \begin{vmatrix} i & j & k \\ 0 & 0 & \Delta t_z \\ 0 & -mg & 0 \end{vmatrix} = (mg \Delta t_z, 0, 0). \]

So we are left with

\[-(mg \Delta t_z, 0, 0) = (d_{M_X}, 0, 0) \]
or looking at just the $x$-coordinates

$$-mg\Delta t_z = d_{MX}$$

$$\Delta t_z = -\frac{d_{MX}}{mg}. \quad (3)$$

So, in order to reduce the DC offset of the $MX$ residual, i.e. to reduce the average forward-backward rocking motions of a walking model, our computation suggests that we should alter the torso center of mass $z$-coordinate by an amount $-d_{MX}/mg$.

2.2.2 Altering the Lumbar Bending Angle

Now we wish to compute an amount $\Delta b$ by which to alter the lumbar bending angle (throughout the entire time interval, not just at the initial time) so that the DC offset for $MX$ is reduced. We can represent the alteration of the lumbar bending angle with the following geometry: consider the triangle consisting of two vectors $r_0$ and $r_1$ with equal length $r_0$ and with a common starting point with an angle $\Delta b$ between them. Suppose the vectors are oriented so that $\Delta l_0$ is drawn in a positive sense (counterclockwise) when it is drawn from $r_0$ to $r_1$. Let $\Delta l = r_1 - r_0$. Assuming $\Delta l_0$ is small, we can apply an approximation from biomechanics which states that the moment arm (lever arm) of a muscle is equal to $\delta l/\delta \theta$ where $\delta l$ is the change in length of the muscle when the joint spanned by the muscle rotates by a small angle $\delta \theta$. Applying this approximation to our triangle, we have that

$$r_0 = \Delta l/\Delta b$$

$$\Delta l = r_0 \Delta b,$$

where $\Delta l = \|\Delta l\|$. We will show how to compute the (direction of) the vector $\Delta l$ later. As before, we define $MX_{new} = r_1 \times mg$ and $MX_{old} = r_0 \times mg$. From the definition of $\Delta l$, we know that $r_1 = r_0 + \Delta l$. Plugging into the equation $MX_{old} - MX_{new} = d_{MX}$, we have

$$r_0 \times mg - (r_0 \times mg + \Delta l \times mg) = (d_{MX}, 0, 0)$$

$$-\Delta l \times mg = (d_{MX}, 0, 0).$$

If we write $\Delta l = (0, \Delta l_y, \Delta l_z)$, then we have

$$\Delta l \times mg = \begin{vmatrix} i & j & k \\ 0 & \Delta l_y & \Delta l_z \\ 0 & -mg & 0 \end{vmatrix} = (mg\Delta l_z, 0, 0).$$

Substituting into the previous equation and extracting just the $x$-coordinates, we have

$$-mg\Delta l_z = d_{MX}.$$ 

Now if we write $\Delta l_z = \Delta l \cos \theta$ where $\theta$ is the angle representing the orientation of the vector $\Delta l$ relative to the positive $y$-axis, and since we know the length of the vector is $\Delta l = r_0 \Delta b$, we have that $\Delta l_z = r_0 \Delta b \sin \theta$, so substituting into the above equation yields

$$-mgr_0 \Delta b \sin \theta = d_{MX}$$

$$\Delta b = -\frac{d_{MX}}{mgr_0 \sin \theta}.$$

The angle $\Delta b$ is the orientation of $r_0$ as measured in a positive (counterclockwise) sense starting from the positive $y$-axis. Since we assumed that $\Delta l_0$ is small, the vector $\Delta l$ is approximately tangent to the circle with radius $r_0$ centered at the pelvis center of mass, i.e. we can assume that $\Delta l$ is just $r_0$ rotated counterclockwise by $90^\circ$ and scaled. Since we defined $\theta$ to be the angle swept counterclockwise from the positive $y$-axis to $\Delta l$, then we can assume that $\theta = b + 90^\circ$. Hence we have

$$\sin \theta = \sin(b + 90^\circ) = \sin b \cos 90^\circ + \cos b \sin 90^\circ = \cos b$$

so

$$\Delta b = -\frac{d_{MX}}{mgr_0 \cos b}. \quad (4)$$

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2.3 Residual Reduction Algorithm (RRA)

The following algorithm will attempt to reduce the $MX$ and $MZ$ DC offsets if it is executed after the inverse kinematics stage of the simulation pipeline and before the CMC stage. We omit implementation details and present only the essential components of RRA here. For our purposes, the inputs needed are:

1. the musculoskeletal model for the subject whose motion is being simulated,
2. the motion file containing the kinematics and ground reaction data for the subject at a finite set of discrete time instants $t_1, t_2, \ldots, t_N$,
3. whether torso x or lumbar extension will be used to reduce the $MX$ DC offset, and
4. whether torso z or lumbar bending will be used to reduce the $MZ$ DC offset.

The first pass of RRA consists of the following steps.

1. Run CMC once on the subject model using the given motion data.
2. Compute the torso x correction amount $\Delta t_x$ using equation 1.
3. Compute the torso z correction amount $\Delta t_z$ using equation 3.
4. Create two arrays $\Delta l_e$ and $\Delta l_b$ that can each hold $N$ real numbers.
5. For each $i = 1, \ldots, N$, compute the lumbar extension correction amount $\Delta l_e[i]$ at the $i$th time step in the motion data using equation 2 and the corresponding value $l_e[i]$ of the model’s lumbar extension angle at that time step.
6. For each $i = 1, \ldots, N$, compute the lumbar bending correction amount $\Delta l_b[i]$ at the $i$th time step in the motion data using equation 4 and the corresponding value $l_b[i]$ of the model’s lumbar bending angle at that time step.
7. Write the numbers $\Delta t_x$ and $\Delta t_z$ and the arrays $\Delta l_e$ and $\Delta l_b$ to a file. If $|\Delta t_x| > 0.1$, set $\Delta t_x$ to zero before writing it to the file. Do the same for $\Delta t_z$. If for any $i$, $|\Delta l_e[i]| > 10^\circ$, then set every entry in the array $\Delta l_e$ to zero before writing its entries to the file. Do the same for the array $\Delta l_b$. The next pass of RRA will automatically apply the values in this file as corrections to the input model and input motion data. So if any correction amount is listed as zero in the file, the result of applying that correction amount to the input model or motion data is the equivalent of making no correction at all. For instance, if $\Delta t_x = 0$, then adding $\Delta t_x$ to the original value of $t_x$ in the second pass of RRA is the same as making no change to the original value of $t_x$.
8. If every correction exceeded the threshold amounts (i.e. if every number in the entire file written in the previous step was zero), then exit with a message saying that this subject data cannot be corrected by RRA.

The second pass of RRA consists of the following steps.

1. Read $\Delta t_x$, $\Delta t_z$, $\Delta l_e$, and $\Delta l_b$ from the file created in the first pass of RRA.
2. If the user chose $t_x$ as the parameter to alter in order to reduce the $MX$ DC offset, add $\Delta t_x$ to the model’s torso x coordinate.
3. If the user chose $t_z$ as the parameter to alter in order to reduce the $MZ$ DC offset, add $\Delta t_z$ to the model’s torso z coordinate.
4. If the user chose $l_e$ as the parameter to alter in order to reduce the $MX$ DC offset, then for each $i = 1, \ldots, N$, add $\Delta l_e[i]$ to $l_e[i]$.
5. If the user chose $l_b$ as the parameter to alter in order to reduce the $MZ$ DC offset, then for each $i = 1, \ldots, N$, add $\Delta l_b[i]$ to $l_b[i]$.
6. Run CMC on the subject model using the given motion data.
References


