Static Optimization
The Inverse Problem

- Use musculoskeletal geometry and assumptions about force distribution to estimate individual muscle forces.
## Key Concepts

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<td>Kinematics</td>
<td>coordinates and their velocities and accelerations</td>
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<td>Kinetics</td>
<td>muscle forces</td>
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<tr>
<td>Muscle physiology</td>
<td>muscle activation-contraction dynamics and force-length-velocity relations</td>
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<tr>
<td>Dynamics</td>
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<td>Musculoskeletal geometry</td>
<td>muscle moment arm</td>
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<td>Optimization</td>
<td>the “distribution” problem</td>
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Kinetics: Muscle Forces

- Kinetics
  - Muscle forces cause the model to accelerate
- Muscle force
  - Applied between origin and insertion points
Muscle Physiology: Muscle Activation-contraction and Force-Length-velocity Relations

- Muscle activation-contraction
  - Biochemical reaction that causes a muscle’s fibers to contract which produces force
Muscle Physiology: Muscle Activation-Contraction and Force-Length-Velocity Relations

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Hof, Human Movement Science, 1984
Muscle Physiology: Muscle Activation-Contraction and Force-Length-Velocity Relations

- Muscle activation-contraction
  - Biochemical reaction that causes a muscle’s fibers to contract which produces force
- Muscle force-length-velocity
  - Force production diminishes for short, long, and fast fibers

```latex
\begin{align*}
F_0^M & = 1.0 \\
F_0^M & = 0.5
\end{align*}
```

```
\begin{align*}
l_0^M & = 1.0 \\
l_0^M & = 0.5
\end{align*}
```

```
\begin{align*}
V_{max} & = 1.0 \\
V_{max} & = 0.5
\end{align*}
```
Musculoskeletal Geometry: Muscle Moment Arm

- Muscle moment arm
  - The perpendicular distance from the line of action of a muscle to the joint center of rotation
  - Transformation from linear force of muscle to angular moment about a joint center

\[ ma_x = \frac{\vec{r} \times \vec{F}}{|\vec{F}|} \cdot \hat{x} \]
Static Optimization

• Determines the “best” set of muscle forces that
  - Produce net joint moments at a discrete time
  - Do not violate muscle force limits
  - Optimize a performance criterion

• Performance criterion attempts to capture the goal of the neural control system
  - Minimize muscle force?
  - Minimize muscle stress?
The Muscle Force Distribution Problem

\[ M_j = \sum_{f=1}^{n_f} F_f r_f - \sum_{e=1}^{n_e} F_e r_e \]

number of flexors \( n_f \)

number of extensors \( n_e \)

Ankle example

\[ M_a = \left( F_{ta} r_{ta} + F_{ed} r_{ed} \right) - \left( F_g r_g + F_s r_s + F_{tp} r_{tp} \right) \]

How can we solve this?

1 equation with \( n_f + n_e \) unknowns
Static Optimization Formulation

minimize \( f(F_m) \) \quad \text{Function of muscle forces}

subject to

\[ M_a(t) = [F_{ta}(t)r_{ta}(t) + F_{ed}(t)r_{ed}(t)] - [F_g(t)r_g(t) + F_s(t)r_s(t) + F_{tp}(t)r_{tp}(t)] \]

\( F_{ta}(t) \leq 900 \text{N} \)
\( F_{ed}(t) \leq 800 \text{N} \)
\( F_g(t) \leq 1500 \text{N} \)
\( F_s(t) \leq 2500 \text{N} \)
\( F_{tp}(t) \leq 1500 \text{N} \)
Example Performance Criteria

\[ f(F_m) = \sum_{m=1}^{nm} F_m \]

\[ f(F_m) = \sum_{m=1}^{nm} \left( \frac{F_m}{PCSA_m} \right)^3 \]

\[ f(F_m) = \sum_{m=1}^{nm} \left( k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2 \]

Muscle force

(Muscle stress)\(^3\) \sim\) Metabolic energy

(Muscle activation)\(^2\)
Example Performance Criteria

\[ f(F_m) = \sum_{m=1}^{nm} F_m \]  
Muscle force

\[ f(F_m) = \sum_{m=1}^{nm} \left( \frac{F_m}{PCSA_m} \right)^3 \]  
(Muscle stress)\(^3\) ~ Metabolic energy

\[ f(F_m) = \sum_{m=1}^{nm} \left( k \frac{F_m}{PCSA_m} \right)^2 \approx \sum_{m=1}^{nm} (a_m)^2 \]  
(Muscle activation)\(^2\)

Possible validations
- Use output to drive a forward dynamic simulation
- Compare qualitatively to experimental EMG
- Compare to measured forces (instrumented hip implant, buckle transducer in tendon)

Difficult to define and validate a good criterion

Major flexors
- Tibialis posterior
- Soleus
- Gastrocnemius
- Major extensors
- Extensor digitorum

Net ankle moment \(M_a\)
Example Performance Criteria

\[ f(F_m) = \sum_{m=1}^{nm} F_m \]

\[ f(F_m) = \sum_{m=1}^{nm} \left( \frac{F_m}{PCSA_m} \right)^3 \]

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Muscle force

(Muscle stress)³ ~ Metabolic energy

(Muscle activation)²

Difficulty to define and validate a good criterion

Hamner and Delp, J. Biomech., 2010
## TIPS & TRICKS

**Inputs:** Can use kinematics from IK or RRA. If using IK, need to filter kinematics

**Residuals:** Add residual actuators to pelvis

**Reserves:** Add reserve torque actuators to trouble-shoot a weak model

**Minimizing residuals & reserves:** Increase maximum control value (default = 1) and lower the maximum force → penalizes activity

**Command Line:** `analyze -S setup_file.xml`
Tracking Simulation
Exercise

1. Given that the rectus femoris muscle has a peak isometric force of 1169 N and it is at its optimal fiber length and zero velocity, what is the force generated for an activation of 0.86?

A. 164 N  
B. 952 N  
C. 1005 N  
D. 1058 N
Exercise

2. For the model shown on the right, which muscle has the largest moment arm about the ankle joint?

A. 1
B. 2
C. Neither (are identical)
Exercise

3. For the model shown on the right, which muscle has the largest moment arm about the knee joint?

A. 1
B. 2
C. Neither (are identical)
Exercise

4. For the model shown on the right, muscle 1 and 2 have the following properties

<table>
<thead>
<tr>
<th>Muscle</th>
<th>Peak Isometric Force (N)</th>
<th>Moment Arm (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>905</td>
<td>3.6</td>
</tr>
<tr>
<td>2</td>
<td>512</td>
<td>3.0</td>
</tr>
</tbody>
</table>

To solve the “distribution” problem minimizing the sum of squared activations, which muscle would be activated more for a given dorsiflexion moment?

A. 1
B. 2
C. Neither (are identical)
Static Optimization in OpenSim

**Inputs**
- Residual or Reserve Actuators
- Kinematics or States
- External Loads
- Exponent \( \min \sum_{m=1}^{nm} (a_m)^p \)
- Idealized actuators or dependent on F-L-V?

**Model**

**Outputs**
- Controls (.xml)
- Activations (.sto)
- Force (.sto)